COUPLINGS TO VECTOR MESONS IN THE RELATIVISTIC QUARK MODEL (*)

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ABSTRACT — Hadron couplings are evaluated using quark graphs and the Bethe-Salpeter formalism. Electromagnetic decays of vector mesons calculated in this way are shown to be consistent with the $\omega - \varphi$ current mixing model. A value $\theta = 33^{o}$ is predicted. Baryon couplings are discussed in the same formalism. When non-relativistic conditions are imposed the model reproduces the usual additivity results with corrections.

1 — INTRODUCTION

Our basic starting points are:

1) Mesons and Baryons are bound state poles in the Bethe-Salpeter (1) $\bar{Q}Q$ and QQQ amplitudes. Whenever q^2, q being the momentum in the $\bar{Q}Q$ or QQQ channel, is close to an on mass shell value the Bethe-Salpeter propagator is saturated by bound state contributions (2). This leads to Vector Meson Dominance and pole dominance of the divergence of the weak axial vector current at quark level (3).

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2) Interactions of hadrons take place via basic quark interactions. This assumption, whose validity is questionable (4), is essentially the one used in the non-relativistic quark-model. It is shown here that the BETHE-SALPETER formalism in general reproduces the non-relativistic quark model additivity results (5, 6, 7, 8). The non relativistic quark model picture of one quark interacting in the presence of the others (spectators) is substituted by a quark triangle graph (9) related to the BETHE-SALPETER normalization equation (10).

Consistency between 1) and 2) was demanded in Ref. [3] to select convenient quark-quark-vector meson (QQV) and quark-quark-pseudoscalar meson (QQP) vertex functions. As an example of the required consistency we imposed, for instance, the condition that the electromagnetic coupling constant f_V of a vector meson determined, as in 1), by saturating the quark electromagnetic current with vector mesons should be compatible with the determination of f_V, as in 2), from quark-antiquark electromagnetic annihilation of the vector meson V. In such a way we arrived at the following vertex functions (3):

QQP:

$$\Gamma_P(p, q) = W(p, q)\gamma^5 G\left[R + \frac{\not q}{2M}\right] \tag{1}$$

QQV:

$$\Gamma_V^{\mu}(p, q) = W(p, q)\,\varepsilon_{\alpha}^{\mu} F\left[\frac{m_V}{2M}\gamma_\alpha - i R\,\sigma_{\alpha\nu}\frac{q^\nu}{m_V}\right] \tag{2}$$

where $m_P(m_V)$ is the meson mass and q its momentum; $M, p \pm q/2$ are the quark mass and momenta; G, R, F and R' are constants; $\varepsilon_{\alpha}^{\mu}$ is the vector meson polarization, $q^{\alpha}\varepsilon_{\alpha}^{\mu} = 0$ when $q^2 = m_V^2$ and $\varepsilon_{\alpha}^{\mu}\varepsilon_{\beta}^{\mu} = q_{\alpha\beta} - q_\alpha q_\beta / m_V^2$; $W(p, q)$ is a form factor satisfying the quark on mass shell condition:

$$W(p, q) = 1 \quad \text{when} \quad \left(p + \frac{q}{2}\right)^2 = \left(p - \frac{q}{2}\right)^2 = M^2 \tag{3}$$

Both the QQP and QQV vertices include derivative and non derivative couplings, pseudovector and pseudoscalar couplings for the QQP interaction and DIRAC and PAULI like couplings for

QQV. The quantities G and R (F and R') can in principle be fixed by solving the BETHE-SALPETER and normalization equations. The vertex functions (1) and (2) have the properties of Llewellyn Smith's Model I ([11]) solution of the BETHE-SALPETER Equation.

General arguments based on the properties of Model I and the BETHE-SALPETER normalization condition, neglecting the potential term ([11]), lead to

$$T_r \left\{ \int \frac{d^4 k}{(2\pi)^4} \left[\overline{\Gamma}(k-q) \frac{i}{\not{k} + \frac{\not{q}}{2} - M} \gamma^0 \frac{i}{\not{k} + \frac{\not{q}}{2} - M} \cdot \right.\right. \tag{4}$$

$$\cdot \Gamma(k, -q) \frac{i}{\not{k} - \frac{\not{q}}{2} - M} - \overline{\Gamma}(k, -q) \frac{i}{\not{k} + \frac{\not{q}}{2} - M} \cdot$$

$$\left.\left. \cdot \Gamma(k, -q) \frac{i}{\not{k} - \frac{\not{q}}{2} - M} \gamma^0 \frac{i}{\not{k} - \frac{\not{q}}{2} - M} \right] \right\} = 4\, q^0$$

where Γ stands for Γ_P or Γ_V^{μ}, supply the relations ([3]):

$$G = F \tag{5}$$

and

$$R = R' \tag{6}$$

The consistency of 1) and 2), as explained above, applied to the couplings to vector mesons further provide ([3]):

$$(\Sigma g_i^V e_i) f_V = F \left(\frac{m_V}{2M} \right) \tag{7}$$

and

$$\Sigma g_i^V e_i K_i = \frac{F R'}{m_V} \frac{1}{f_V} \tag{8}$$

where e_i is the charge of quark Q_i ($i = p, n, \lambda$) in units of electron charge; g_i^V is the CLEBSCH-GORDON coefficient giving the $Q_i \overline{Q}_i$ contribution to the vector meson V and K_i is the anomalous magnetic moment of quark Q_i in units of the quark mass. To relate the values of F and R' to known quantities we write (7) for the particular case of the ρ meson:

$$F\left(\frac{m_\rho}{2M}\right)=\frac{f_\rho}{\sqrt{2}} \tag{9}$$

and combine (8) with the non-relativistic quark model result of the equality of the magnetid moments of the quark and the proton, which we shall re-derive later, $K_P \simeq 2.79/2\,m_{\text{proton}}$, to obtain (5):

$$R' \simeq \frac{m_\rho}{2M} \tag{10}$$

We remark that in the formalism of the BETHE-SALPETER equation the free quark must have a large mass (experimentally it is contrained to $M \geq 5$ GeV) and this has the consequence that R' and R are small, of order (m_ρ/M), and that the magnetic moment of the quark is mostly anomalous.

We do not discuss in the present paper couplings to pseudoscalar mesons. They are evaluated using the vertex function (1). As (1) does not show a dependence on the meson mass our results agree with exact SU(3) and the non relativistic quark model. The fact that from (10) and (6) R is negligible small justifies the use of a pure pseudovector coupling, as in non relativistic quark model (12), at the QQP vertex.

The vector meson mass factors present in the QQV vertex function (2) are effectivelly SU(3) breaking factors that will affect hadronic couplings to vector mesons.

In Sec. 2 we discuss the electromagnetic decay of vector mesons and show the parallelism of the quark model and current mixing model descriptions of $\omega - \varphi$ mixing. We predict for the mixing angle the value, $\theta \simeq 33°$. In Sec. 3 we use the BETHE-SALPETER formalism and the vertex function (2) to evaluate PPV and VVP coupling constants. In Sec. 4 the BETHE-SALPETER formalism for baryon couplings is developed. Corrections are introduced to the usual SU(6) non-relativistic quark model predictions of baryon magnetic moments.

2 — ELECTROMAGNETIC DECAY OF VECTOR MESONS AND THE FIRST WEINBERG SUM RULE

The constraint of Vector Meson Dominance to the electromagnetic interaction of quarks in the limit $q^2 \to 0$ gives, from (7),

$$f_\rho^{-1} : f_\varphi^{-1} : f_\omega^{-1} = \frac{1}{m_\rho} : \frac{\cos\theta}{\sqrt{3}\, m_\varphi} : -\frac{\sin\theta}{\sqrt{3}\, m_\omega} \tag{11}$$

where θ is the $\omega - \varphi$ mixing angle. One notes that the first WEINBERG SUM RULE [13] applied to the isopin and hypercharge currents and saturated by the observed vector mesons [14,15],

$$\frac{m_\rho^2}{f_\rho^2} = 3\left[\frac{m_\varphi^2}{f_\varphi^2} + \frac{m_\omega^2}{f_\omega^2}\right] \tag{12}$$

is identically satisfied by the above relations (11). We will now exploit the SU(3) properties of the electromagnetic current and in this way we are able to rederive (12).

In the present model the breaking of SU(3) is related to the dynamics of the $Q\overline{Q}$ interaction in the BETHE-SALPETER equation and it appears explicitly in the mass factors of the QQV vertex functions. When we deal with free quarks we expect to find in SU(3) a good symmetry, for instance, quark mass breaking factors being negligible compared to the quark mass. The same applies to the electromagnetic current which is assumed bilinear in the quark and antiquark fields.

$$J^\mu = J_I^\mu + J_Y^\mu \tag{13}$$

where

$$J_I^\mu = \frac{1}{2}\left(\overline{\psi}_P \gamma^\mu \psi_P - \overline{\psi}_n \gamma^\mu \psi_n\right) \tag{14}$$

is the isopin current, and

$$J_Y^\mu = \frac{1}{6}\left(\overline{\psi}_P \gamma^\mu \psi_P + \overline{\psi}_n \gamma^\mu \psi_n - 2\,\overline{\psi}_\lambda \gamma^\mu \psi_\lambda\right) \tag{15}$$

is the hypercharge current. J^{μ} is a U spin scalar and its matrix element for a transition from the vacuum to a U spin vector $\bar{Q}Q$ state must then vanish, i. e.:

$$\underset{I}{<}\bar{Q}Q|J^{\mu}|0> = \sqrt{3}\underset{Y}{<}\bar{Q}Q|J^{\mu}|0> \tag{16}$$

with

$$|\bar{Q}Q\underset{I}{>} = \frac{1}{\sqrt{2}}[|\bar{p}p> - |\bar{n}n>]$$

and

$$|\bar{Q}Q\underset{Y}{>} = \frac{1}{\sqrt{6}}[|\bar{p}\bar{p}> + |\bar{n}n> - 2|\bar{\lambda}\lambda>].$$

We consider (16) in the limit $q^2 \to 0$ and recall, from (2) and (7) that

$$\Gamma^{\mu}_{V}(q^2 = 0) \sim m_V \tag{17}$$

$$f_V \sim m_V \tag{18}$$

Saturating (16) with the physical ρ meson and an unphysical Y meson, isosinglet member of an SU(3) octet, we obtain, with (17) and (18)

$$\frac{m_\rho}{f_\rho} = \sqrt{3}\,\frac{m_Y}{f_Y} \tag{19}$$

The right hand side of (16) can also be saturated by the physical mesons φ and ω. Their quark content is related to $|\bar{Q}Q\underset{Y}{>}$ by the equation

$$|\bar{Q}Q\underset{Y}{>} = \cos\theta\,|\bar{Q}Q\underset{\varphi}{>} - \sin\theta\,|\bar{Q}Q\underset{\omega}{>} \tag{20}$$

where $\cos\theta$ and $\sin\theta$ are the usual $\omega - \varphi$ mixing coefficients. Eq. (16) then gives

$$\frac{m_\rho}{f_\rho} = \sqrt{3}\left[\frac{m_\varphi}{f_\varphi}\cos\theta - \frac{m_\omega}{f_\omega}\sin\theta\right]. \tag{21}$$

This equation combined with (11) reproduces (12).

One should notice that in this approximation of vector meson dominance Eqs. (19) and (21) could equally well be derived from

(16) taking the point of view of CURRENT-FIELD Identities [16], i. e., using the following operator relations between currents and fields:

$$J_I^{\mu} = \frac{m_\rho^2}{f_\rho} \rho^{\mu} \tag{22}$$

$$J_Y^{\mu} = \frac{m_Y^2}{f_Y} Y^{\mu} = \frac{1}{f_Y} [m_\varphi^2 \cos \theta_Y \varphi^{\mu} - m_\omega^2 \operatorname{sen} \theta_Y \omega^{\mu}] \tag{23}$$

with

$$f_Y = f_\varphi \cos \theta_Y = - f_\omega \sin \theta_Y . \tag{24}$$

From (12) and (24) we have

$$\tan \theta = \frac{m_\omega}{m_\varphi} \tan \theta_Y \tag{25}$$

a relation which is typical of an $\omega - \varphi$ current mixing model [16, 17]. In a mass mixing model the state mixing angle θ and the hypercharge current $\omega - \varphi$ decomposition angle θ_Y are the same.

From the point of view of the quark model $\omega - \varphi$ mixing results from the combination of singlet and octet SU(3) states while in the current mixing model it is related to the presence of off diagonal elements in the propagator matrix, an explicit reference to states never being required. However COLEMAN and SCHNITZER [17] have shown that it is in fact possible to use the language of states and mixing of states through an angle θ inside the framework of the current mixing model provided the octet and singlet unperturbed masses are equal. This is the normal assumption not only in the quark model but also in the current mixing model [18]. Thus our quark model description of the current in the vector meson dominance approximation is equivalent to the current mixing model. To derive expressions for θ_Y or θ we combine (12), (19) and (24) to obtain

$$m_Y^2 = m_\varphi^2 \cos^2 \theta_Y + m_\omega^2 \sin^2 \theta_Y \tag{26}$$

or from (25),

$$\frac{1}{m_Y^2} = \frac{1}{m} \cos^2 \theta + \frac{1}{m_\omega^2} \sin^2 \theta \tag{27}$$

The knowledge of the $\bar{Q}Q$ binding potential should give via the BETHE-SALPETER equation a relation between the mass of the vector mesons and their quark content. As we do not know the potential we make use of the well established GELL-MAN OKUBO mass formula relating masses of particles belonging to the same SU(3) representation. It gives for the mass of the isosinglet:

$$m_Y^2 = \frac{1}{3}(4\,m_k^2 - m_\rho^2) \tag{28}$$

Using (28) in (26) and (27) we calculate

$$\theta_Y \simeq 40^\circ$$

and

$$\theta \simeq 33^\circ .$$

In any mixing model the angles θ and θ_Y are related quantities but experimentally they can be independently measured using for θ_Y

$$\frac{\Gamma(\omega \to e^+ e^-)}{\Gamma(\varphi \to e^+ e^-)} = \left(\frac{m_\omega}{m_\varphi}\right) \tan^2 \theta_Y \tag{29}$$

and for θ,

$$\frac{f^{(0)}_{kk\varphi}}{f_\varphi} = \frac{1}{2}\cos^2\theta . \tag{30}$$

Experiments give [19] from (29), $\theta_Y \simeq 41^\circ$ and from (30), $\theta \simeq 31^\circ$. In both cases the agreement is reasonable, thus giving support to relation (25) and the current msxing model. It is interesting to remark that Eq. (26) is true at the same time in the current and mass mixing models [15, 18]. However (27) is only satisfied with current mixing.

For our predicted $\theta = 33^\circ$ Eqs. (11) give:

$$f_\rho^{-2} : f_\varphi^{-2} : f_\omega^{-2} = 9 : 1.19 : 0{,}85$$

to be compared with the experimental averages [20] $9 : 1{,}5 \pm 0.4 : 1{,}1 \pm 0.3$. The relations (11) can also be used to compute f_Y^2. From (24),

$$\frac{1}{f_Y^2} = \frac{1}{f_\varphi^2} + \frac{1}{f_\omega^2} \tag{31}$$

giving

$$\frac{f_Y^2}{4\pi} = 3\,\frac{1}{\left(\frac{m_\rho}{m_\varphi}\right)^2 \cos^2\theta + \left(\frac{m_\rho}{m_\omega}\right)^2 \sin^2\theta}\;\frac{f_\rho^2}{4\pi} . \tag{32}$$

Only in the case of mass degeneracy does (32) reproduce the SU(3) limiting value $f_Y^2/_{4\pi} = 3 f_\rho^2/_{4\pi}$; for the observed masses it imposes

$$\frac{f_Y^2}{4\pi} > 3\,\frac{f_\rho^2}{4\pi}. \tag{33}$$

Using in (31) the averaged data on f_ω^2 and f_φ^2 one obtains $f_Y^2/_{4\pi} \simeq 7.2 \pm 1.2$ which is consistent with (33) $[3 f_\rho^2/_{4\pi} = 6.0]$ but does not exclude the SU(3) value. Introducing the value $\theta = 33°$ in (32) we obtain $f_Y^2/_{4\pi} = 8.8$.

3 — THREE-MESON VERTICES

In this section we rederive some results of Ref. [3] but concentrate here on the influence of the mixing angle θ. We start by considering *PPV* interactions and illustrate graphically in Fig. 1 the equation for the $\rho^0\pi^+\pi^-$ vertex. Γ_π and Γ_ρ are the

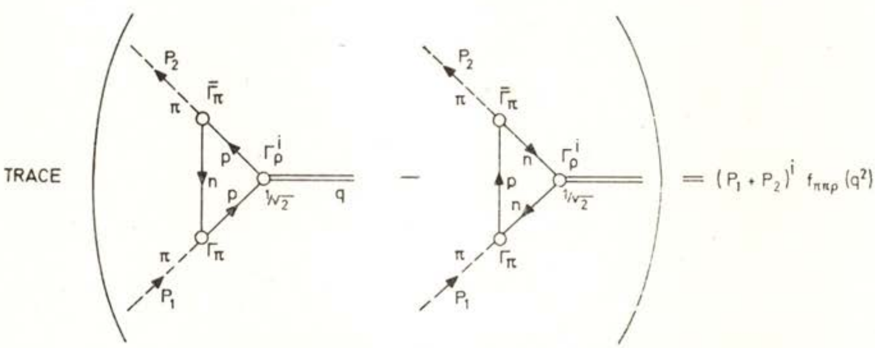

Fig. 1 — Quark graph calculation of the $\rho\pi\pi$ coupling constant.

$QQ\pi$ and $QQ\rho$ vertex functions as in (1) and (2). The only coupling in Γ_ρ^i that contributes to the trace calculation is the DIRAC coupling $\gamma^i F m_\rho/2M$. If the form factor $W(p, q)$ in Γ_ρ does not drastically modify the result of the integration, the equations of Fig. 1 in the limit $q^2 \to 0$, becomes the BETHE-SALPETER normalisation equation (4), apart a factor $f_{\rho\pi\pi}^{(0)}/f_\rho$, and provided F is given by Eq. (9). Thus

$$f_{\rho\pi\pi}^{(0)} \simeq f_\rho. \tag{34}$$

The equation of Fig. 1 may be expanded as

$$\frac{m_\rho}{\sqrt{2}}\left[A+B\left(\frac{m_\pi}{M}\right)+C\left(\frac{m_\pi}{M}\right)^2+\cdots\right]- \tag{35}$$

$$-\frac{m_\rho}{\sqrt{2}}\left[A-B\left(\frac{m_\pi}{M}\right)+C\left(\frac{m_\pi}{M}\right)^2+\cdots\right]=\left(\frac{m_\pi}{M}\right)f^{(0)}_{\rho\pi\pi}.$$

With the approximation $(m_{\pi/M})^2, (m_{\rho/M})^2 << 1$ both in the quark propagators and form factors, A, B and C become constants independent of the masses of the interacting mesons. A and C vanish because of the $q^2, (q\cdot p), p^2$ dependence of $W(p, q)$ imposed from general symmetry properties of the BETHE-SALPETER wave function for pseudoscalar and vector mesons (11). With only the B term left in (35) the $f^{(0)}_{PPV}$ couplings are proportional to the vector meson mass and are independent of the pseudoscalar meson masses. For the $\varphi K^+ K^-$ ($\varphi \overline{K}^0 K^0$) and $\omega K^+ K^-$ vertices for instance we obtain

$$f_{KK\varphi}=\left(\sqrt{\frac{3}{2}}\cos\theta\right)\frac{1}{\sqrt{2}}\left(\frac{m_\varphi}{m_\rho}\right)f^{(0)}_{\rho\pi\pi} \tag{36}$$

$$f_{KK\omega}=\left(-\sqrt{\frac{3}{2}}\sin\theta\right)\frac{1}{\sqrt{2}}\left(\frac{m_\omega}{m_\rho}\right)f^{(0)}_{\rho\pi\pi} \tag{37}$$

In the aproximation $f^{(0)}_{PPV} \approx f_{PPV}(m_V^2)$ and for $\theta = 33°$ we predict the ratios $\Gamma(V \to PP)/\Gamma(\rho \to \pi\pi)$ shown in Table I.

TABLE I

Ratios $\Gamma(V \to PP)/\Gamma(\rho \to \pi\pi)$

Decay	Theoretical	Experimental
$\phi - K^+ K^-$	0.0230	0.0168 ± 0.0033
$\phi - K^{-0} K^0$	0.0148	0.0128 ± 0.0026

From Eqs. (36) and (37) we again find consistency with the current mixing model, for instance in the relation

$$\frac{f^{(0)}_{KK\varphi}}{f^{(0)}_{KK\omega}} = - \frac{m_\varphi}{m_\omega} \frac{1}{\tan \theta} . \tag{38}$$

In the present model the PVV couplings can be worked out in a similar way [3]. One obtains for the $f_{\rho\omega\pi}$ coupling the relation

$$\frac{f^2_{\omega\rho\pi}\, m^2_\rho}{\left[\sqrt{\frac{2}{3}} \cos \theta + \frac{1}{\sqrt{3}} \sin \theta\right]^2} \simeq 4 f^2_{\rho\pi\pi} \tag{39}$$

which, for ideal mixing, reproduces the $SU(6)_w$ result [21]. The VVP couplings are proportional to $1/m^2_V$ and, when the experiment angle is used in the decays involving the φ meson, this factor helps to bring down the $\varphi \to \pi\gamma, \eta\gamma$ rates as seems to be experimentally required [22,23]. Our results on the VVP couplings are in approximate quantitative agreement with the ones of Ref. [22] and we refer to it for experimental comparison.

4 — COUPLINGS OF BARYONS

In previous work we have shown [3] how Llewellyn Smith's Model I combined with vector meson dominance and the GOLDBERGER-TREIMAN relation at quark level is able to reproduce the non-relativistic quark model results relating quark to hadron coupling constants. These relations were originally derived using the idea of additivity of the interactions of quarks inside the baryons in a kind of shell model of the baryon. We show now how additivity works for the baryons in the present model and how we are able to recover basic additivity results of the non-relativistic quark model.

If the baryon wave functions are solutions of the three quark channel homogeneous BETHE-SALPETER equation, they satisfy a BETHE-SALPETER normalisation equation. This is shown in Fig. 2, and the symbols are a generalisation of the notation of Ref. [11].

In particular represents the three-body wave function,

$$\chi\left(P;\frac{P}{3}+k_1,\frac{P}{3}+k_2,\frac{P}{3}+k_3\right)$$

with $P^2=M_B^2$ and $k_1+k_2+k_3=0$. Again we neglect the potential contributions to normalisation. Note that Eq. of Fig. 2 is a very formal one. We will disregard complications due to dis-

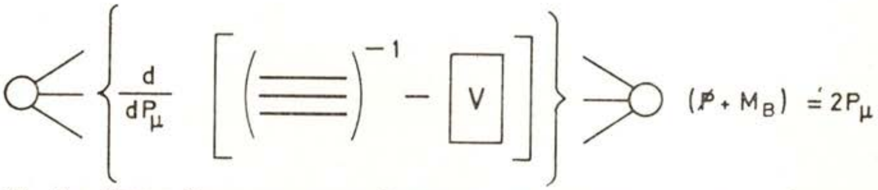

Fig. 2 — Bethe-Salpeter normalisation equation for baryons. The notation for Wave functions, propagators and generalised potentials is as in Ref. 3.

connected potential terms since we are neglecting the contribution of the potentials altogether in the normalisation.

The BBV couplings in the limit $q^2\to 0$, neglecting as before the QQV form factor, are described by the same integrals as the normalisation equation, but with different coupling constants. For the electric coupling of the nucleon we obtain the vector meson dominance result

$$g^{(0)}_{\rho NN}\simeq\frac{1}{2}f_\rho \tag{40}$$

and, in the ideal mixing limit

$$g^{(0)}_{\omega NN}\simeq 3\left(\frac{1}{2}f_\rho\right). \tag{41}$$

This is just a reflection of the fact that vector dominance connects baryonic to electric charge for both nucleons and quarks. We also obtain, using (11), the relation for the saturation of the electromagnetic current by the ρ and ω poles,

$$\frac{g^{(0)}_{\rho PP}}{f_\rho}+\frac{g^{(0)}_{\omega PP}}{f_\omega}=1. \tag{42}$$

As the normalisation condition is defined at zero momentum transfer, $q = 0$, the only model independent statements one can make are on the electric couplings. Some further assumptions are required to evaluate the magnetic coupling, the weak axial vector decay coupling and the couplings to pseudoscalar mesons, whose matrix elements are linear in q.

In a picture of the baryons interacting through the quarks we will suppose that when one quark interacts the other two, »spectators», behave in an averaged sense as a single diquark object. Inside the baryons the quarks move with small space momenta so one expects only s wave interactions. Thus to have the octet and decuplet baryons in a *56* representation of SU(6) the interacting quarks must couple to appropriate combinations of SU(3) triplet (T) and sextet (S) diquarks, the triplet having spin-zero and the sextet spin 1[24]. These average spectator diquarks can then be described by a scalar field for the triplet and a pseudovector field for the sextet.

The introduction of these diquarks has the advantage of reducing the three-body problem to a two-body one. The coupling $BQQQ$ may then be written as a combination of the couplings BQT (scalar coupling) and BQS (pseudovector coupling). If $\hat{O}_B$ is a baryon level operator and $\hat{O}_Q$ the corresponding quark level one, the matrix element $< B | \hat{O}_B | B >$ is evaluated by the graphical equation of Fig. 3, where $C_i^{T(S)}$ are products of SU(3) and

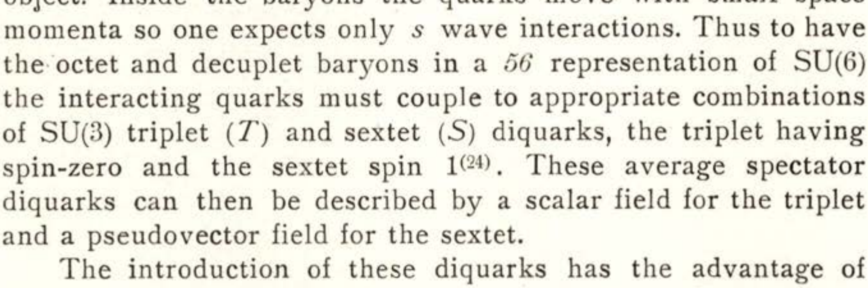

Fig. 3 — Quark graphs for the calculation of baryon couplings.

SU(6) coefficients, and, apart from form factors which are supposed to be the same by SU(6), Γ^T and Γ_j^S are given by:

$$\Gamma^T = 1 \quad \text{and} \quad \Gamma_j^S = \frac{1}{\sqrt{3}} \gamma^5 \gamma_j$$

the factor $1/\sqrt{3}$ in Γ_j^S being a spin weighting factor. The coefficients C^T and C^S are easily determined by writing the SU(6) baryon wave function in a quark-diquark model[24] for the proton, for instance, we have

$$P \equiv \frac{1}{\sqrt{2}}\left[-\sqrt{\frac{2}{3}}\, S_1 n + \sqrt{\frac{1}{3}}\, S_2 p\right] + \frac{1}{\sqrt{2}}\, T_1 p \qquad (43)$$

where S_1, S_2 are sextet and, T_1 a triplet non-interacting diquark, and n, p the interacting quark (*). Multiplying the square of these coefficients by 3, since each quark can interact, we get

$$\begin{aligned} C_n^T &= 0\ , \quad & C_n^S &= 1 \\ C_p^T &= \frac{3}{2}, \quad & C_p^S &= \frac{1}{2}\cdot \end{aligned} \qquad (44)$$

For the neutron case p and n are interchanged in (43) and (44).

We evaluate the graphs of Fig. 3 in the Breit frame of the baryon, and neglect all contributions from P' in the numerator an from the timelike component of the S diquark. In the case of the magnetic moment calculations the quark operator will be

$$\hat{O}_i^{\mu} \equiv e_i [F_1(q^2)\gamma^{\mu} - i K_i F_2(q^2)\sigma^{\mu\nu} q_\nu] \qquad (45)$$

where $F_1(q^2=0) = F_2(q^2=0) = 1$. The terms linear in q give the magnetic moment and both, γ^j and $\sigma^{j\nu}$ contribute. The quark anomalous magnetic moment gives a contribution $K_i \sigma^{j\nu} q_\nu$ that multiplies the normalisation integral which we write schematically in the form

$$I_0 = \int H(P, P', q=0)\ (P_0' + M)^2 d^4 P' = 1 \qquad (46)$$

where form factors and denominators have been absorbed in $H(P, P', q)$ and $(P_0' + M)^2$ comes from the interacting quark propagator numerators. The electric coupling γ^j gives two contributions, one coming from the q term in the quark propagators

(*) $S_1 \equiv [pp]$, $S_2 \equiv \frac{1}{\sqrt{2}}[pn + np]$, $T_1 \equiv \frac{1}{\sqrt{2}}[pn - np]$.

of the form $\frac{1}{2M}\sigma^{j\nu}q^{\nu}$ which contains as a factor the integral

$$I_1 = \int H(P, P', q=0)\,(P'_0 + M)\,2\,M\,d^4 P' \qquad (47)$$

and another form the q dependence of the nucleon spinors of the form $\frac{1}{2M_N}\sigma^{j\nu}q^{\nu}$ with a factor

$$I_2 = \int H(P, P', q=0)\,(P'^2_0 - M^2)\,d^4 P' \qquad (48)$$

From (46), (47) and (48) we have

$$I_0 = I_1 + I_2 = 1\,. \qquad (49)$$

Going further into a non-relativistic situation we assume now that the average value of P'^2_0 in I_2 equals that of M^2, which means that the interacting quark acts effectively as if on its mass shell. This assumption is consistent with using in (45) only the on mass shell couplings. Neglecting integrals in P' and equating integrals in P'^2_0 to integrals in M^2 we reproduce the conditions for a non-relativistic model with quarks essentially at rest and static interactions. Eq. (49) then gives

$$I_2 = 0 \qquad \text{and} \qquad I_0 = I_1 = 1\,.$$

The equation of Fig. 3 gives then the non-relativistic SU(6) quark model results for the magnetic moments, in the form,

$$\sum_{\nu = n, p, \lambda} \left\{ -\frac{1}{3} \sum_S C^S_\nu + \sum_T C^T_\nu \right\} e_\nu\, \mu_\nu = e\, \mu_B \qquad (50)$$

where the coefficients $-\frac{1}{3}$ and 1 arise from the commutation of the Γ^j_S and γ^i or $\sigma^{i\nu}$. We note that contributions from the integral I_2 or from timelike diquark components would have spoiled the additivity rule expressed by (50). In particular for the proton we have,

$$\mu_{\text{proton}} \simeq \frac{1}{2M} + K_p\,. \qquad (51)$$

The baryon axial vector weak couplings and pseudovector couplings to pseudoscalar mesons are similar to the magnetic couplings and the additivity results are derived in the same way.

In the present model the magnetic moment of the n and λ quark differ from that of the p quark. From (8) and for angles θ close to the ideal mixing angle $\theta_i \simeq 35^\circ$ such that $\cos(\theta - \theta_i) \approx 1$ and $\sin(\theta - \theta_i) \approx 0$, we derive

$$K_n = K_p\,[1 + \beta] \tag{52}$$

and

$$K_\lambda = K_p\,[1 - \alpha] \tag{53}$$

where

$$\beta = \frac{1 - \sqrt{3}\sin\theta\left(\frac{m_\rho}{m_\omega}\right)^2}{1 + \frac{\sin\theta}{\sqrt{3}}\left(\frac{m^\rho}{m_\omega}\right)^2} \tag{54}$$

and

$$\alpha = \frac{1 + \frac{1}{\sqrt{3}}\sin\theta\left(\frac{m_\rho}{m_\omega}\right)^2 - \frac{4}{3}\sqrt{\frac{3}{2}}\left(\frac{m_\omega}{m_\varphi}\right)^2\cos\theta}{1 + \frac{\sin\theta}{\sqrt{3}}\left(\frac{m_\rho}{m_\omega}\right)^2}\,. \tag{55}$$

For $\theta = 33^\circ$ we find $\alpha = 0.40$ and $\beta = 0.07$. The factors α and β provide corrections to the usual non-relativistic calculations of magnetic moments. In the limit of quarks with only anomalous magnetic moment the effect of the corrections α and β is shown in Table II (Experimental data from Ref. [25]). In all cases the corrections act in the right direction and strongly reduce the magnetic moments of the Λ and Ω.

In this model other corrections could equally well have been considered, such as violations of $SU(6)$ in the wave functions and contributions from I_2 integrals, and we have no a priori reason to neglect them compared to the α and β corrections.

TABLE II

Magnetic Moments of Baryons ($\mu_{\text{proton}} = 2.792$)

Baryon	Theoretical				Experimental
	No Correction $K_\lambda = K_n = K_p$		Correction α and β		
	μ/μ_{proton}	μ	μ/μ_{proton}	μ	
N	$-2/3$	-1.86	$-2/3\left(\dfrac{1+\frac{2}{3}\beta}{1+\frac{1}{9}\beta}\right)$	-1.93	-1.913
Λ	$-1/3$	-0.93	$-1/3\left(\dfrac{1-\alpha}{1+\frac{1}{9}\beta}\right)$	-0.57	-0.73 ± 0.16
Σ^+	1	2.793	$\dfrac{1-\frac{1}{9}\alpha}{1+\frac{1}{9}\beta}$	2.66	2.5 ± 0.5
Ω	-1	-2.793	$-\dfrac{1-\alpha}{1+\frac{1}{9}\beta}$	-1.71	?

5 — CONCLUSIONS

In this paper we extended the work of Ref. [3] in two directions to include:

1) A discussion of the relation between the relativistic quark model treatment of the electromagnetic interactions in the region $q^2 \approx 0$ and the approach from CURRENT-FIELD Identities with particular emphasis on the problem of $\omega - \phi$ mixing. Our model agrees with the current mixing model, and the predicted value for the mixing angle $\theta \approx 33^\circ$ agrees with experiment. This angle was used to evaluate meson coupling constants and baryon magnetic moments.

2) The development of the BETHE-SALPETER formalism to calculate baryon couplings. As expected in a three body problem we find a complicated picture in comparison to the naive quark model additivity. Our formulae contain too many unknown parameters to have, in general, a predictive power. However, when the conditions for a mon-relativistic situation are introduced, our formalism reproduces in that limit the generally successful results of the non-relativistic quark model.

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