ON DYNAMICS OF ELEMENTARY PARTICLES DERIVED FROM RELATIVISTIC QUARK MODELS (*)

J. DIAS DE DEUS
Department of Physics
University College London

ABSTRACT — The interactions of hadrons are described from the point of view of quarks.

Weak, electromagnetic and strong coupling constants are evaluated with a relativistic quark model using the BETHE-SALPETER formalism. Uniquely defined S U (3) mass breaking factors are derived from the quark dynamics inside the mesons.

A dualized approach to the classic quark model connecting the low and high energy predictions is proposed and a simple model for the four body amplitude satisfying duality and the additive quark model is applied in the determination of meson-baryon low energy parameters. Analogous calculations are also shown for the hypothetical quark-meson process.

Quark duality diagrams are used to constrain dual resonance amplitudes. One of these constraints takes the form of a superconvergent relation to be saturated in local mass regions. These relations are studied for various processes both in the forward and backward direction.

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CHAPITRE 1

Quarks and quark models

Quarks were introduced in particle physics in 1964 by Gell-Mann [1] and Zweig [2] on the grounds of purely group theoretical arguments. With a basic SU(3) triplet one can generate the other SU(3) representations by direct products of the $\underline{3}$ (quark) and $\overline{3}$ (antiquark) representations. Baryons are generated from combinations

$$\underline{3} \times \underline{3} \times \underline{3} = \underline{1} + \underline{8} + \underline{8} + \underline{10} \qquad \text{(I. 1)}$$

and mesons from the product

$$\underline{3} \times \overline{3} = \underline{1} + \underline{8} \qquad \text{(I. 2)}$$

More representations for baryons and mesons can be obtained by including extra $\underline{3} \times \overline{3}$ products. However the most striking quark model result is perhaps the spectacular dominance of the representations indicated in (I. 1) and (I. 2) over all the other possible ones.

The invention of quarks immediately raised the question of their existence. Much effort was invested in the search for quarks and contradictory claims — pro and against their existence — were recently made [3]. The question of existence of quarks is still open and the answer has to be left to the experimentalists.

Theoretically one can distinguish two lines of thought with which the quark model is approached: a «mathematical quarks» school, restraining quarks to the role of carriers of SU(3) quantum numbers, and a «realistic quarks» school trying to give quarks the usual properties of physical particles. The difference between the two schools is easily seen making use of Lipkin's image of the two planes in strong interaction physics, the (I, Y, B) plane of internal symmetries and Mandelstam's (s, t, u) plane of space-time physics. The «mathematical quarks» school wants to leave quarks in the (I, B, Y) plane where they were born, while the «realistic quarks» school aims to bring them to the (s, t, u) plane. In this thesis we take the realistic quarks point of view.

An interesting and not unrelated question is the problem of hierarchy: what is the place of quarks in the hierarchy of elementary particles? Are quarks «aristocratic» particles or do we still have a «democratic» world in strong interactions? In the «mathematical quarks» school quarks are definitely aristocrats: they do not even materialize in the (s, t, u) plane as all the other particles. In the «realistic quarks» school if one simply regards quarks as the ultimate constitutive blocks of hadronic matter they certainly remain «aristocratic». However one is not necessarily led to that attitude: a quark can be, for instance, seen as «made up» of a meson and another quark, and, in general, one could think of an enlarged bootstrap scheme where quarks and hadrons would appear as a result of the presence of quark and hadronic forces. As far as strong interactions are concerned quarks can be treated in the same footing as the other particles and thus we favour a «democratic» solution à la Chew (4).

The realistic quark model was introduced by Morpurgo (5) and extensively developed by Dalitz (6). The first problems the model had to discuss were related to the quark mass and the forces binding quarks inside hadrons. The suggested model of quarks with large mass and a flat bottomed potential, extracted from an intuitive picture of hadrons as compound particles, is still a basic starting point for realistic versions of quark model.

It should be noticed that in the mathematical quarks approach it makes sense to talk about the mass of the quark to the extent that this quantity has an SU(3) meaning. An isospin doublet-singlet quark mass difference, for instance, can be introduced to generate SU(3) mass formulae. A definition of an absolute quark mass is of course outside the scope of SU(3). In the realistic approach is frequently defined with an effective quark mass M^* (bound quark mass) and a free quark mass M. It is the latter which corresponds to the usual definition of mass for physical particles and thus is the one we shall use. The relation between the two concepts of mass was discussed by Lipkin and Tavkhelidze (7) who showed that they might be made compatible for certain types of potentials if one thinks of the effective mass M^* as a quantity including M and the effect of the potential well, i. e.

$$M^* = M - V \tag{I. 3}$$

The effective mass concept is related to the interpretation of quarks as quasi particles in analogy with phonons, i. e., they only exist while constituents of hadrons (8). Such models being still realistic avoid the difficulty of quarks not having been unmistakably found. For most of the purposes, because of (I. 3), the effective mass and the free mass approaches, are equivalent. There are however interesting differences in the two treatments. One occurs with electromagnetic interactions. In the quasiparticle treatment the quarks are considered as Dirac particles (zero anomalous magnetic moment). In the free quark treatment the magnetic moment must be dominantly anomalous. If quarks happen to exist this is in fact a prediction of this class of models. Another problem where the two treatments differ is in the comparison of high energy additivity predictions. Accepting that reactions should be compared at the same quark center of mass energy, when comparing cross-sections for the reactions $AB \to AB$ and $AB' \to AB'$ one should take values of laboratory momenta P_B and $P_{B'}$ such that,

$$P_{B'}/P_B = Z_{B'}/Z_B \tag{I. 4}$$

in the effective mass (and momentum) approach where $Z_B(Z_{B'})$ is the number of quarks in $B(B')$ (9), and

$$P_{B'}/P_B = m_{B'}/m_B \tag{I. 5}$$

in the free quark mass treatment where $m_B(m_{B'})$ is the mass of particle $B(B')$ (10). Prescription (I. 4) holds in meson-baryon scattering ($p\pi \to p\pi$ compared to $pK \to pK$) but seems to be violated when comparing meson baryon to baryon baryon scattering ($p\pi \to p\pi$ compared to $pp \to pp$). The validity of either (I. 4) or (I. 5) or the failure of both will possibly in future be tested more easily in inclusive processes. If in the reaction $A+B \to C+X$

(X being «anything») the collision of A with B is effectively a collision of the quarks of A with the quarks of B the production cross-section of C must show forward-backward symmetry when plotted against the longitudinal momentum of C in the quark center of mass frame. The existing data is at relatively low energy and one should wait for much higher energy data. However some experiments favour already the existence of such frame and prescription (I. 4)([11]). If these results are confirmed and either (I. 4) or (I. 5) is proven to be correct that would become a remarkable success of the realistic quark model.

The nature of the potential responsible for the binding of quarks is a problem for which no answer was found. In the bootstrap perspective, the binding forces should have an origin in the usual particles rather than in some special mechanism. Phenomenological potentials have been used in nonrelativistic([12]) (Schrödinger) and relativistic([13]) (Bethe-Salpeter) dynamical equations, the most successful ones being of harmonic oscillator type. The various resonances of a given trajectory are described in terms of orbital quark excitations. This simple and intuitively appealing models are rather powerful particulariy in the classification of baryonic resonances([14]).

It should be remarked that a priori the dynamics of three quarks in baryons could well be completely different from the dynamics of the two quarks inside mesons. In principle there is no reason to expect similar orbital excitations and trajectories with approximate universal slope, as seems to be the case. Thus it is of great interest to consider models for the structure of hadrons, in which the constraint of universality of the Regge trajectories is imposed from the beginning. Such models were inspired by duality and the Veneziano model([15]). In one of them, Susskind's model([16]), the hadronic matter is described as a continuum (a string, a rubber band) and instead of quark orbital excitations we have now vibrations of this continuum. As the continuum is the same for all hadrons the spectrum of resonances is also universal (same slope of trajectories and same daughter structure). The quarks are not the fundamental dynamical objects but are simply relevant singularities embedded in the continuum, playing the role of boundary conditions. A quark could be in this model represented in the same was as any other particle, a continuum with just one singularity, having the same type of excitations (quark trajectory). These crude ideas applied to meson-

-meson and meson-barion scattering led in an elegant way to the Veneziano formula. Conceptually these models are rather atractive but attempts [17] to describe details of the resonance spectrum, namely baryon resonances, failed to recover the good results of the more conventional SU(6) quark model with orbital excitations.

Applied to hadron interactions the usual quark model [18] gives two kinds of predictions. At low energy it predicts coupling constants and widths of resonances. At high energy it predicts relations between cross-sections. Normally these predictions appear as if they were independent. However, having in mind the ideas of duality, it is natural to think of models giving simultaneously results in both low and high energy regions and being able to relate them. No satisfactory models of this type exist yet. In this thesis we use a relativistic (Bethe-Salpeter) approach to quark model to describe two body decays and coupling constants (Chapter II and Refs. [19] and [20]). This approach is our main tool in the low energy region. At high energy we rely on the usual additivity results. We then write simple models for the amplitudes containing these two pieces of information supplied by the quark model and show that such models satisfy duality (Chapter III and Ref. [21]). Dual resonance models [22] are the natural generalization of these attempts, but the existing dual relativistic models [23,24] are not unfortunately of much practical use. In this thesis we limit ourselves on deriving from the quark underlying structure constraints to the scattering amplitude to be satisfied in dual resonance models (Chapter IV and Ref. [25]).

CHAPTER II

The Bethe-Salpeter approach to relativistic quark models

1. Introduction.

The non-relativistic quark model has been a very successful model in predicting decay widths and coupling constants in hadron interactions [26, 27, 28]. However, when decays involving quark--antiquark annihilation are considered the model runs into trouble requiring the inclusion of SU(3) non-invariant space wave functions.

Discussing the problem of the structure of the meson wave functions from a relativistic point of view Llewellyn Smith [29] found that the wave functions that give good predictions in annihilation processes and preserve SU(3) are not the ones that correspond to the weak binding limit of the non-relativistic quark model. Thus the necessity of a relativistic treatment of quark model and of the use of wave functions with a relativistic structure.

The fact that one uses wave functions with a relativistic structure does not mean that the quarks have relativistic internal motion. In all calculations we always have the freedom of fixing the average values of the internal momenta. Regarding the realistic quark model the conclusion from our work is that its basic idea that quarks move with small space momenta within hadrons must be kept. This requires a deep flat bottomed potential with a long range force and massive quarks [5, 6] ($M \gtrsim 5\, GeV$).

In this chapter we discuss a Bethe-Salpeter [30] type of approach to quark model. We make use of the Bethe-Salpeter formalism in two ways:

(*i*) Mesons and baryons (orbital $L = 0$ states) are considered as bound state poles in the Bethe-Salpeter $\bar{Q}Q$ (antiquark--quark) and QQQ (3 quark) amplitudes. Whenever P^2, P being the momentum in the $\bar{Q}Q$ or QQQ channel, is close to an on mass shell value the Bethe-Salpeter propagator is saturated by bound state contributions [31]. This leads to Vector Meson Dominance and pole dominance of the divergence of the weak axial vector current at quark level [19].

(*ii*) Interactions of hadrons are supposed to take place via basic quark interactions. The non relativistic quark model picture of one quark interacting in the presence of the others (spectators) is substituted by a quark triangle graph [32] related to the Bethe--Salpeter normalization equation [33].

Consistency between (*i*) and (*ii*) is demanded as a criterion to select convenient quark-quark-vector meson (QQV) and quark-quark-pseudoscalar meson (QQP) vertex functions. Such consistency leads to Llewellyn Smith's preferred relativistic model [29] (Model I).

The wave functions for mesons used here were suggested from the solutions of the $\bar{Q}Q$ Bethe-Salpeter equation with a separable potential. We assume that their form is more general than the separable potential itself. Thus we do not work strictly in the separable potential model, this simple model being occasionally used only for orders of magnitude estimates.

In the QQP and QQV vertex functions we include derivative and non derivative couplings. The referred consistency between (*i*) and (*ii*) determines effective SU(3) mass breaking factors in the various QQP and QQV coupling constants. Because of (*ii*) these mass breaking factors will affect hadron coupling constants to pseudoscalar and vector mesons. In general our results for coupling constants improve the SU(3) symmetry limit and the non relativistic quark model.

In Section 2 we briefly describe the Llewellyn Smith Bethe--Salpeter formalism applied to quarks. In Section 3, we discuss possible models and use the criterion of consistency between (*i*) and (*ii*) to select the correct one. In Section 4 three particle coupling constants, including baryon couplings, are evaluated in our formalism. We end, Section 5, with a short discussion on the existing relativistic quark model approaches to the four legged processes as an introduction to chapter III.

2. The Bethe-Salpeter formalism and Llewellyn Smith Models.

To describe quark-antiquark processes we need the basic ingredients of the Bethe-Salpeter formalism for fermion-antifermion scattering. We are not interested in the full two-body propagator (given by the inhomogeneous equation) but only in the pole terms caused by the presence of bound state mesons (given by the homogeneous equation). In the notation we fellow Ref. [29].

If $\psi_\alpha(x_1)$ and $\overline{\psi}_\beta(x_2)$ are the quark and antiquark Heisenberg field operators the meson wave function in coordinate space is defined by

$$\chi_{\alpha\beta}(X, x) = < 0 \,|\, T(\psi^1_\alpha(x_1)\,\overline{\psi}^2_\beta(x_2)) \,|\, B > \tag{II. 1}$$

where $X = (x_1 + x_2)/2$ is the four dimension center of mass vector and $x = x_1 - x_2$ is the relative distance between particles 1 and 2. In momentum space we have

$$\chi_{\alpha\beta}(P, q) = \int e^{iP.X + iqP.x}\, \chi_{\alpha\beta}(X, x) \frac{d^4 x}{(2\,\pi)^4} \tag{II. 2}$$

where now P is the quark-antiquark overall momentum and q the relative momentum. The conjugate wave function is defined by imposing the Feynman boundary conditions. In the complex q_0 plane $\chi(P, q)$ shows structure along the real axis, a left hand cut from $-\sqrt{q^2 + M^2} + \frac{P^0}{2}$ and a right hand one from $\sqrt{q^2 + M^2} - \frac{P_0}{2}$. $\chi(P, q)$ and the conjugate wave function $\overline{\chi}(P, q)$ are defined as limits of an analytic function evaluated respectively above the right hand cut and below the left hand one. They are related by:

$$\overline{\chi}(P, q) = \gamma^0 \chi^+ (P, q)\, \gamma^0 . \tag{II. 3}$$

For convenience we shall later use the vertex function $\Gamma(P, q)$ related to $\chi(P, q)$ by:

$$\Gamma(P,q) = \left(\not{q} + \frac{\not{P}}{2} - M\right) \chi(P,q) \left(\not{q} - \frac{\not{P}}{2} - M\right). \qquad \text{(II. 4)}$$

For fermion-fermion scattering χ has 16 components and transforms as the outer product of a spinor and a conjugate spinor. We write it in a matrix form: four 4×4 matrices corresponding to the positive component-positive component $(++)$ term in the product of spinors, the $(+-)$, $(-+)$ and the $(--)$ terms. In the rest frame of the bound state meson one thus has:

$$\chi(m_B, \underset{\sim}{0}; q) = \begin{pmatrix} \chi^{+-} & \chi^{++} \\ \chi^{--} & \chi^{-+} \end{pmatrix} \qquad \text{(II. 4)}$$

In the limit of free quark and antiquark the following relations between the components of χ hold:

$$\chi^{+-} \simeq \chi^{-+} \simeq \frac{\underset{\sim}{\sigma} \cdot \underset{\sim}{q}}{M} \chi^{++} \qquad \text{(II. 5)}$$

$$\chi^{--} \simeq \frac{\underset{\sim}{q}^2}{M^2} \chi^{++} \qquad \text{(II. 6)}$$

When the relative motion is negligible, e. g.,

$$\underset{\sim}{q}^2 \approx 0 \qquad \text{(II. 7)}$$

from (II. 5) and (II. 6) only χ^{++} is important and in particular

$$\chi^{--} << \chi^{++} \qquad \text{(II. 8)}$$

Equation (II. 7) is rather natural in the framework of quark model and has a strong intuitive appeal. However when combined with (II. 6) leads to (II. 8) and this last relation is in the origin of the troubles in annihilation processes. If one accepts the weak binding limit model, Equations (II. 5) and (II. 6), the Weisskopf-Van Royen paradox [28] and space wave functions with large SU(3) breaking cannot be avoided, unless one abandons the attractive limit $\underset{\sim}{q}^2 \approx 0$. Thus the real paradox of the non relativistic model of having to give up the non relativistic approximation to the internal quark motion to save its contradictions. Llewellyn Smith's analysis (29) showed clearly that the $q\!\!\!/^2 \simeq 0$ approximation can be kept (it is in fact basic to derive SU(6)) because it only implies Equation (II. 8) and SU(3) breaking of the wave functions if Equation (II. 6) is valid. Certainly the weak binding limit is not a sensible approximation in quark model and Equation (II. 6) is not reliable.

Without making a priori approximations in $\chi(P, q)$ one can study its properties under symmetry operations and derive the general structure of the wave functions. This was done by Llewellyn Smith (29) and we only give here, as an example, the structure of the wave function for a pseudoscalar meson:

$$\chi = \gamma_5 (A + q \cdot P B q\!\!\!/ + C P\!\!\!/ + D (q\!\!\!/ P\!\!\!/ - P\!\!\!/ q\!\!\!/)) \qquad \text{(II. 9)}$$

where A, B, C and D are even functions of $(q \cdot P)$.

To extend the analysis of possible models for the wave functions we consider the homogeneous Bethe-Salpeter equation:

$$\left(q\!\!\!/ - \frac{P\!\!\!/}{2} - M\right) \chi(P, q) \left(q\!\!\!/ + \frac{P\!\!\!/}{2} - M\right) = - \int \hat{V}(P, q, k)\, \chi(P, k) \frac{d^4 k}{(2\pi)^4} \qquad \text{(II. 10)}$$

where $\hat{V}$ is the potential operator. If $\chi(P, q)$ is a solution of (II. 10) it satisfies a normalization equation:

$$\begin{aligned} i \operatorname{Tr} \int \frac{d^4 q}{(2\pi)^4} \Big\{ & (2\pi)^8 \bar{\chi}(P, q) \left[\frac{\partial}{\partial P_\mu} \left(q\!\!\!/ - \frac{P\!\!\!/}{2} - M \right) \right] \chi(P, q) \left(q\!\!\!/ + \frac{P\!\!\!/}{2} - M \right) \\ & + (2\pi)^8 \bar{\chi}(P, q) \left(q\!\!\!/ - \frac{P\!\!\!/}{2} - M \right) \chi(P, q) \left[\frac{\partial}{\partial P_\mu} \left(q\!\!\!/ + \frac{P\!\!\!/}{2} - M \right) \right] \qquad \text{(II. 11)} \\ & + \int d^4 k\, \bar{\chi}(P, q) \frac{\partial \hat{V}(P, q, k)}{\partial P_\mu} \chi(P, q) \Big\} = 2 P_\mu . \end{aligned}$$

In his discussion of the wave functions Llewellyn Smith makes two important assumptions:

1) The potential $\hat{V}$ is SU(3) invariant, e. g., it is weakly dependent on the meson masses. This assumption allows one via the Bethe-Salpeter normalization equation to relate wave functions for different mesons.

2) A smooth extrapolation in the meson masses is valid. This means that in taking the limit $(m_B/M) \to 0$, natural in a world of massive quarks, the physics described by (II. 10) is not drastically changed.

With these two assumptions and using the space reflection symmetry properties of A, B, C and D (Equation (II. 9)) in (II. 11) Llewellin Smith formulates his Models. The characteristics of the Models are shown in Table II. 1.

TABLE II. 1

Model	Characteristics
I	$\chi^{++} \simeq \chi^{--}$ $(\chi^{++} - \chi^{--})/m_P = \text{const.}$
II	$\chi^{++} \simeq -\chi^{--}$ $\chi^{++} - \chi^{--} = \text{const.}$
III	$\chi^{++} >> \chi^{--}$ $\chi^{++}/\sqrt{m_P} = \text{const.}$

Model I is the one favoured by Llewellin Smith. It has the following interesting properties:

1) The wave functions are, up to corrections of order $(M_B/M)^2$, SU(3) invariant.

2) Making the approximation $(q/M)^2 << 1$ one obtains approximate SU(6) as in the non relativistic model.

3) As the dominant component of the wave function is meson mass independent it does not change drastically in the soft pion limit $P_\mu \to 0$ and thus this limit can be safely taken.

4) It requires in the limit $(m_B/M) \to 0$ the validity of the Goldstein equation [34] as an eigenvalue equation and this, on the other hand implies a potential less singular at the origin than the one particle exchange. This is a constraint found in all quark model calculations, non relativistic [35] and relativistic ones [36].

5) Applied to the evaluation of decays of mesons it provides the relations

$$f_\rho^{-2} : f_\varphi^{-2} : f_\omega^{-2} = \frac{1}{m_\rho^2} : \frac{\cos^2\theta}{3\,m_\varphi^2} : \frac{\sin^2\theta}{3\,m_\omega^2} \qquad \text{(II. 12)}$$

for the electromagnetic annihilation of vector mesons and

$$F_\pi = F_K \qquad \text{(II. 13))}$$

for the weak two body decays of pseudoscalar mesons. Both predictions are in fair agreement with experiment. Models II and III, the last one corresponds to the weak binding limit, give bad predictions. This point will be further discussed later.

In the next section we will study simple models embodying the properties of Llewellyn Smith general Models I, II and III.

The description of the mesons as bound states in the $Q\bar{Q}$ channel is not equivalent to saying that the mesons are completely determined in teir behaviour by their quark content. If however we are prepared to accept this we can largely extend the field of application of the Bethe-Salpeter approach. The idea then becomes to reduce all hadronic interactions to quark interactions. In general we will thus suppose that the interactions of hadrons take place through the quark currents and that the interactions of quarks are mediated by hadron fields.

In particular it was shown ([31]) that in the Bethe-Salpeter formalism applied to quarks vector meson dominance and the Goldberger-Treiman relation appear as a result of the saturation of the quark-antiquark propagator by meson bound state contributions. These are expected to dominate when the meson is near its mass shell. Thus from the point of view ot quarks the electromagnetic decay of vector mesons (or the weak axial vector two body pseudoscalar meson decay) will be described by the graph of Fig. 1a ([29]). Neglecting three-body interaction the three-meson vextex will be predominantly given by a triangle (renormalized) graph (Fig. 1b), which may be justifiably evaluated as a Feynman integral ([32]). But the four legged process of Fig. 1c cannot be treated as a Feynman graph because there will be large contributions from poles in some channels. Only some kind of duality interpretation is adequate. Baryon interactions will similarly be decomposed into their quark constituents.

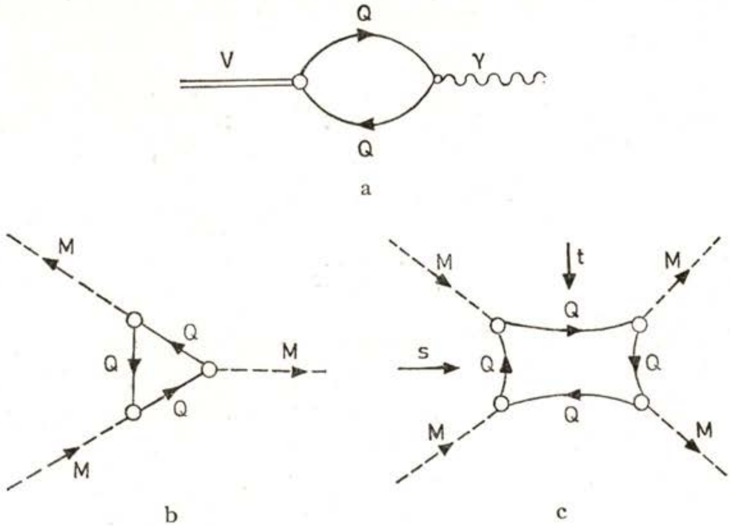


Fig. 1

3. Models for QQV and QQP vertex functions.

In this Section we introduce quark-quark-pseudoscalar meson (QQP) and quark-quark-vector meson (QQV) vertex functions which are simple realizations of Llewellyn Smith's Models I, II and III. They have the structure of solutions of the $Q\bar{Q}$ homogenous Bethe-Salpeter equation with a separable potential [29] and this simple model is occasionally used for orders of magnitude estimates. Both the QQP and QQV vertices include derivative and non-derivative couplings, pseudovector and pseudoscalar couplings for the QQP interaction and Dirac and Pauli-like couplings for QQV. We write the vertex functions as:

QQP:

$$\Gamma_P(P,p) = W(P,p)\,\gamma^5\, G\left[\left(\frac{m_P}{2M}\right)^s R - \left(\frac{m_P}{2M}\right)^v \frac{\not{P}}{2M}\right] \qquad \text{(II. 14)}$$

QQV:

$$\Gamma^{\mu}_V(P,p) = W(P,p)\,\varepsilon^{\mu}_{\alpha}\, F\left[\left(\frac{m_V}{2M}\right)^e \gamma_\alpha + iR'\left(\frac{m_V}{2M}\right)^m \sigma_{\alpha\nu}\frac{P^\nu}{m_V}\right] \qquad \text{(II. 15)}$$

where $m_P(m_V)$ is the meson mass and P its moments; $M, p \pm \frac{P}{2}$ are the quark mass and momenta; G, R, F and R' are Model dependent constants and s, v, e and m are parameters which characterize the Models; $\varepsilon^{\mu}_{\alpha}$ is the vector meson polarization, $P_\alpha \varepsilon^{\mu}_{\alpha} = 0$ when $P^2 = m_V^2$ and $\varepsilon^{\mu}_{\alpha} \varepsilon^{\nu}_{\beta} = g_{\alpha\beta} - q_\alpha q_\beta / m_V^2$; $W(P,p)$ is a form factor required for the convergence of some integrals and satisfies the quark on mass shell condition:

$$W(P,p) = 1 \quad \text{when} \quad \left(p + \frac{P}{2}\right)^2 = \left(p - \frac{P}{2}\right)^2 = M^2 .$$

The quantities G and R (F and R') can in principle be determined by solving the Bethe-Salpeter and normalization equations. Table II. 2 shows in the different Models the values of the parmeters s, v, e and m.

TABLE II. 2

Model	Vertex Function Parameters			
	QQP		QQV	
	s	v	e	m
I	0	0	1	0
II	1	—1	0	1
III	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

We now compute for the various models [29], the neutral vector meson electromagnetic coupling constants f_V, using the graph of (Fig. 1a), and similarly the pseudoscalar meson annihilation parameter F_P. The result, easily obtained with the vertex functions (II. 14) and (II. 15), can be written in the form:

$$\frac{1}{f_V} = \left[\sum g_i^V e_i \Big/ \left(\frac{m_V}{2M}\right)^k \right] \mathrm{I} \qquad \text{(II. 16)}$$

and

$$F_P = \left[2\, M g_A \Big/ \left(\frac{m_P}{2M} \right)^l \right] \mathrm{I}' \qquad \text{(II. 17)}$$

where g_A is the ratio of axial to vector coupling for quarks;

e_i is the charge of quark Q_i $(i = \mathscr{P}, \mathfrak{N}, \lambda)$ in units of electron charge;

g_i^V is the Clebsh-Gordan coefficient giving the $Q_i \overline{Q}_i$ contribution to the vector meson V;

k, l are numbers depending on the chosen Model;

and I, I' are the loop integrals which in the approximation $\frac{m_P}{M}, \frac{m_V}{M} << 1$ are independent of the meson masses. The quark axial vector coupling renormalization constant and quark and nucleon Cabibbo angles have been absorbed in I'. The loop integrals I and I' have not necessarily the same values for the various Models.

In Table II. 3 are shown the parameters k and l in the different cases as well as the mass dependence of f_V and f_P. The experimental ratios of f_V's or F_P's for different vector or pseudoscalar mesons strongly favour Model I (29).

TABLE II. 3

Model	Annihilations			
	Pseudoscalar Mesons		Vector Mesons	
	Parameter l	m_P dependence of F_P	Parameter k	m_V dependence of f_V
I	0	const.	1	m_V
II	1	$1/m_P$	2	m_V^2
III	$\frac{1}{2}$	$1\ m_P^{1/2}$	3/2	$m_V^{3/2}$

Let us now take the electromagnetic interaction of quarks in the limit $P^2 \to 0$ and consider first the Dirac coupling. In that limit this coupling gives the quark electric charge. As $P^2 = 0$ is not far from the vector meson poles it is reasonable to assume that the quark electromagnetic current is dominated by vector mesons. Graphically and making use of Fig. 1a we have Fig. 2. The corresponding equations are, from (II. 15) and (II. 16):

$$\sum g_i^V e_i = F \left(\frac{m_V}{2M} \right)^e \frac{1}{f_V} \qquad \text{(II. 18-a)}$$

$$= \left(\sum g_i^V e_i \right) \left(\frac{m_V}{2M} \right)^{e-k} F I . \qquad \text{(II. 18-b)}$$

To avoid contradictions between the left and the right hand sides

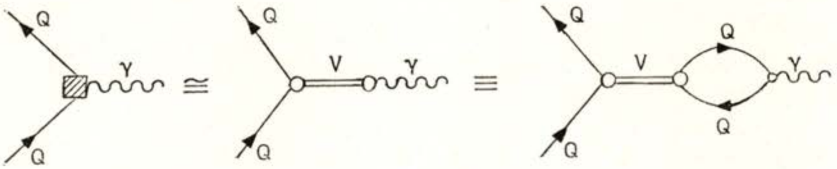


Fig. 2

of (II. 18-b) in the sense that no dependence on vector meson masses should occur in the right hand side we require

$$e = k \qquad \text{(II. 19)}$$

and this constraint is only satisfied by Model I. To satisfy Equation (II. 18-b) one still needs to ensure

$$F I = 1 \qquad \text{(II. 20)}$$

but this is only a constraint on the form factor for the QQV vertex. For Model I the Dirac coupling is then, in the limit $P^2 \to 0$, consistent with a vector meson dominance description of the quark electromagnetic interaction.

We treat in a similar way and in the same limit the divergence of the weak axial vector interaction of quarks (Fig. 3) with the resulting equations:

$$2\,M g_A = \sqrt{2}\, g_Q F_\pi \tag{II. 21-a}$$

$$= (2\,M g_A) \left(\frac{m_P}{2\,M}\right)^{-l} \sqrt{2}\, g_Q\, I' \tag{II. 21-b}$$

where

$$g_Q = \frac{1}{\sqrt{2}}\, G \left(\frac{m_P}{2\,M}\right)^{v} \left(\left(\frac{m_P}{2\,M}\right)^{s-v} R + 1 \right) \tag{II. 22}$$

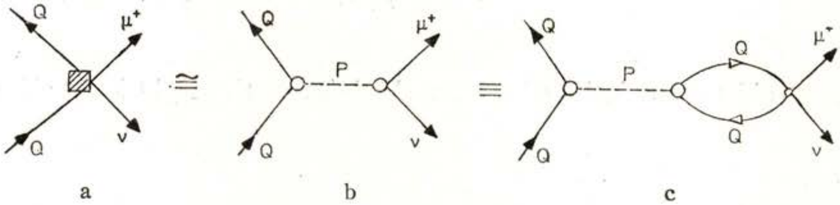


Fig. 3

is the overall $Q\,Q\,\pi^0$ coupling constant. If we assume negligible mass dependence of the last factor in (II. 22), which is strictly true in Model I and is justified in the other Models if R is not too large then the consistency of (II. 21-b) requires:

$$v = l$$

a condition that is satisfied only in Model I. The additional constraint is:

$$\sqrt{2}\, g_Q\, I' = 1\,. \tag{II. 24}$$

Equation (II. 21-a) contains nothing but a Goldberger-Treiman relation for quarks.

It is remarkable that only Model I satisfies vector meson dominance and *PCAC* for quarks [37]. It is also the only Model that predicts a *KSFR* type relation [38]:

$$f_\rho F_\pi = c\, m_\rho \tag{II. 25}$$

where in this model, from (II. 18-a), (II. 21-a), (II. 23) and (II. 24)

$$c = \sqrt{2}\, g_A I'/I = g_A F/g_Q .$$

We shall use the experimental determination of c to relate g_A and g_Q [39]. From now on we restrict ourselves to Model I ($s = v = 0$; $e = 1, m = 0$; $l = 0, k = 1$) and the vertex functions

$$\Gamma_P(P, p) = W(P, p)\, \gamma^5\, G \left[R - \frac{P\!\!\!/}{2M} \right] \qquad \text{(II. 26)}$$

$$\Gamma_V^{\mu}(P, p) = W(P, p)\, \varepsilon_{\alpha}^{\mu}\, F \left[\left(\frac{m_V}{2M} \right) \gamma_{\alpha} + i R' \sigma_{\alpha\nu} \frac{P^{\nu}}{m_V} \right] \qquad \text{(II. 27)}$$

Applying vector meson dominance to the Pauli coupling, one obtains

$$\sum e_i g_i^V K_i = \frac{F R'}{m_V} \frac{1}{f_V} \qquad \text{(II. 28)}$$

where K_i gives the anomalous contribution to the magnetic moment of the quark in units of the quark mass. From relations (II. 28) and with a mixing angle θ close to the ideal mixing angle we have approximately:

$$K_{\mathcal{N}} = 2 \frac{1 - \dfrac{\sin\theta}{\sqrt{3}} \left(\dfrac{m_{\rho}}{m_{\omega}} \right)^2}{1 + \dfrac{\sin\theta}{\sqrt{3}} \left(\dfrac{m_{\rho}}{m_{\omega}} \right)^2} K_{\mathcal{P}} \qquad \text{(II. 29)}$$

and

$$K_{\lambda} = \frac{4}{3} \frac{\sqrt{\dfrac{3}{2}} \left(\dfrac{m_{\rho}}{m_{\varphi}} \right)^2 \cos\theta}{1 + \dfrac{\sin\theta}{\sqrt{3}} \left(\dfrac{m_{\rho}}{m_{\omega}} \right)^2} K_{\mathcal{P}} . \qquad \text{(II. 30)}$$

For ideal mixing of octet and singlet SU(3) states ($\varphi = \lambda \bar{\lambda}$) and with $m_{\rho}^2 \approx m_{\omega}^2$, (II. 29) and (II. 30) give

$$K_{\mathcal{N}} = K_{\mathcal{P}}, \;\; K_{\lambda} = \left(\frac{m_{\rho}}{m_{\varphi}} \right)^2 K_{\mathcal{P}} . \qquad \text{(II. 31)}$$

The first relation is the usual non-relativistic quark model assumption, the second differs by the factor $(m_\rho/m_\varphi)^2$.

As we mentioned before G and R (F and R') can be determined by solving the Bethe-Salpeter and normalization equations but to obtain such a solution a knowledge of the interacting $Q\bar{Q}$ potential and form factor $W(P,p)$ is required. As we do not know these we use general arguments to relate G to F and R to R'.

Llewellyn Smith showed that in the case of a spin independent potential the pseudoscalar and vector meson relativistic wave functions for vanishing $\left(\frac{m_P}{M}\right)$ and $\left(\frac{m_V}{M}\right)$ are related by SU (6). This implies, in our model,

$$R\,G = R'\,F \tag{II. 32}$$

a result which we take as more fundamental than the form of the potential. On the other hand the normalization equation (II. 11) written in the rest frame of the meson becomes

$$\begin{aligned}\mathrm{Tr}\Bigg\{\int \frac{d^4 k}{(2\pi)^4}\Bigg[&\bar{\Gamma}(P,k)\frac{i}{\not k+\frac{\not P}{2}-M}\gamma^0\frac{i}{\not k-\frac{\not P}{2}-M}\cdot\\ &\cdot\Gamma(P,k)\frac{i}{\not k+\frac{\not P}{2}-M}-\bar{\Gamma}(P,k)\frac{i}{\not k-\frac{\not P}{2}-M}\Gamma(P,k)\cdot\\ &\frac{i}{\not k+\frac{\not P}{2}-M}\gamma^0\frac{i}{\not k+\frac{\not P}{2}-M}\Bigg]\Bigg\}=4\,P^0\end{aligned} \tag{II. 33}$$

where Γ stands for Γ_P or Γ_V^μ. The potential contributions have been neglected as in Ref. [29]. Working out the trace calculations for P and V mesons under the assumption $<k^2> \lesssim M^2$ and comparing the results

$$(a\,R^2 + b\,R)\,G^2 = (a\,R'^2 + b\,R')\,F^2 \tag{II. 34}$$

where, in the approximation $m_V^2, m_P^2 << M^2$, a and b are integrals independent of the meson masses. Equations (II. 32) and (II. 34) then give

$$G = F \tag{II. 35}$$

and

$$R = R' \qquad \text{(II. 36)}$$

We note that the separable potential model in general predicts R to be of the order of R' but a precise relation, like (II. 36) is model dependent. It also gives, for a dominantly scalar interaction $R < 1$ and positive.

To find an estimate for R and R' we use the non-relativistic quark model result of the equality of the magnetic moments of the quark and the proton, which we shall rederive later, $K_{\mathscr{P}} = 2.79/2\, m_{\text{proton}}$. Equation (II. 18-*a*) for the ρ meson becomes:

$$F \frac{m_\rho}{2M} = \frac{f_\rho}{\sqrt{2}} \qquad \text{(II. 37)}$$

and combined with (II. 28) gives

$$R' = \left(\frac{2.79}{2\, m_{\text{proton}}} m_\rho\right) \frac{m_\rho}{2M} \simeq \frac{m_\rho}{2M} \qquad \text{(II. 38)}$$

where the last expression is an approximate numerical result. It is worth recalling that R' does not depend on m_V but (II. 38) shows that R' (and R) is small of order (m_ρ/M). With (II. 38) the QQV coupling satisfies broken $\mathrm{SU}(6)_W$ universality [37].

Neglecting R, compared to 1, we derive from (II. 21-*a*), (II. 22), (II. 25), (II. 35), (II. 36), and (II. 37) expressions for the basic quark level weak axial vector coupling constant g_A and pseudovector $QQ\pi$ coupling constant (g_Q/M) in terms of known quantities:

$$g_A = \frac{c}{\sqrt{2}} \qquad \text{(II. 39)}$$

and

$$\left(\frac{g_Q}{M}\right) = \left(\frac{f_\rho}{m_\rho}\right). \qquad \text{(II. 40)}$$

Equation (II. 40) is an additivity quark model result. In (II. 39) with the experimental value of c, $c \simeq 1$, g_A agrees with the additivity result for axial weak decays for baryons, $g_A \simeq 0.70$ [18].

We have shown that Llewellyn Smith's Model I contains PCAC and vector meson dominance consistently built in. The model is then dynamically consistent with a description of the meson interactions through the splitting of the mesons into the quarks and the rebuilding of the mesons from the quarks.

4. Three body coupling constants.

In Ref. [19] we developed the techniques for evaluating coupling constants involving three mesons in the Bethe-Salpeter formalism (triangle graph of Fig. 1-b). Llewellyn Smith Model I mass factors in the QQV vertex function act effectively as SU(3) mass breaking factors and we obtain the results (19):

$$f_{PPV} \propto m_V \tag{II. 41}$$

$$f_{VVP} \propto 1/m_V^2 \tag{II. 42}$$

The PPV and VVP coupling constants are related by:

$$\frac{f^2_{\omega\rho\pi}\, m^2_\rho}{\left[\sqrt{\frac{2}{3}}\cos\theta + \frac{1}{\sqrt{3}}\sin\theta\right]^2} \simeq 4\, f^2_{\rho\pi\pi} \tag{II. 43}$$

wich reproduces the $SU(6)_W$ result of Sakita and Wali (40) in the ideal mixing limit. In [19] and [20] the effect of these mass factors in the widths of resonances was shown in various examples. Numerical results depend in many cases on the value taken for the $\omega - \varphi$ mixing angle and this makes the comparison of (II. 41) and (II. 42) with the SU(3) symmetric limit (no mass breaking factors) and experiment sometimes ambiguous. There are however at least two cases where prescriptions (II. 41) and (II. 42) seem to be tetter than the SU(3) limit. One is in the $K^* K \pi$ coupling constant (no $\omega - \varphi$ mixing involved). The other is in the $\pi\gamma, \eta\gamma$ decays of the meson φ. If we take the ideal mixing angle ($\theta \simeq 35°$) the decay is forbidden in the planar graph approximation, if however we take $\theta < 35°$ (experimentally $\theta \simeq 33°$ (41)) without the $1/m_\varphi$ mass factor of (II. 42) the calculated width may become too large. In table II. 4 we show for these two cases our predictions compared to the SU(3) limit and experiment.

TABLE II. 4

Decay	Width (MeV)		
	Prediction	SU (3) limit	Experiment
$K^* \to K\pi$	49.6	36.2	50.6 ± 0.7
$\Phi \to \pi\gamma$	0.0052	0.0070	< 0.012

For further discussion of meson couplings and connections to vector meson dominance we refer to [19] and [20]. In this section we would like to concentrate mainly in the problem of baryon couplings to mesons.

We have shown in Section 2 how Llewellyn Smith's Model I combined with vector meson dominance and the Goldberger-Treiman relation at quark level is able to reproduce the non-relativistic quark model results relating quark to hadron coupling constants. These relations were originally derived using the idea of additivity of the interactions of quarks inside the baryons in a kind of shell model of the baryon. We show now how additivity works for the baryons in the present model and how we are able to recover basic additivity results of the non-relativistic quark model.

If the baryon wave functions are solutions of the three quark channel homogeneous Bethe-Salpeter equation, they satisfy a Bethe-Salpeter normalization equation. This is shown in Fig. 4,

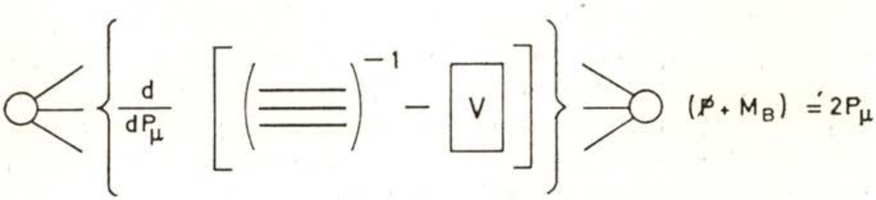
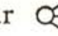


Fig. 4

and the symbols are a generalization of the notation of Ref. [29]. In particular  represents the three-body wave function,

$$\chi\left(P; \frac{P}{3} + k_1, \frac{P}{3} + k_2, \frac{P}{3} + k_3\right)$$

with $P^2 = M_B^2$ and $k_1 + k_2 + k_3 = 0$. Again we neglect the potential contributions to normalization. Note that Equation of Fig. 4 is a very formal one. We will disregard complications due to disconnected potential terms since we are neglecting the contribution of the potentials altogether in the normalization.

The BBV couplings in the limit $q_V^2 \to 0$, neglecting as in Ref. [19] the QQV form factor, are described by the same integrals as the normalization equation, but with differənt coupling constants. For the electric coupling of the nucleon we obtain the vector meson dominance result

$$g_{\varrho NN}(0) \simeq \frac{1}{2} f_\varrho \tag{II. 44}$$

and, in the ideal mixing limit

$$g_{\omega NN}(0) \simeq 3 \left(\frac{1}{2} f_\varrho \right), \tag{II. 45}$$

This is just a reflection of the fact that vector dominance connects baryonic to electric charge for both nucleons and quarks. We also obtain, using (II. 12), the relation for the saturation of the electromagnetic current by the ϱ and ω poles,

$$\frac{g_{\varrho pp}(0)}{f_\varrho} + \frac{g_{\omega pp}(0)}{f_\omega} = 1. \tag{II. 46}$$

As the normalization condition is defined at zero momentum transfer, $q = 0$, the only model independent statements one can make are on the electric couplings. Some further assumptions are required to evaluate the magnetic coupling, the weak axial vector decay coupling and the couplings to pseudoscalar mesons, whose matrix elements are linear in q.

In a picture of the baryons interacting through the quarks we will suppose that when one quark interacts the other two, «spectators», behave in an average sense as a single diquark object. Inside the baryons the quarks move with small space momenta so one expects only S wave interactions. Thus to have the octet and decuplet baryons in a $\underline{56}$ representation of SU(6) the interacting quarks must couple to appropriate combinations of SU(3) triplet (T) and sextet (S) diquarks, the triplet having spin-zero and the sextet spin 1 ([42]). These average spectator diquarks can then be described by a scalar field for the triplet and a pseudovector field for the sextet.

The introduction of these diquarks has the advantage of reducing the three-body problem to a two-body one. The coupling $BQQQ$ may then be written as a combination of the couplings BQT (scalar coupling) and BQS (pseudovector coupling). If $\hat{O}_B$ is a baryon level operator and $\hat{O}_Q$ the corresponding quark level one, the matrix element $< B | \hat{O}_B | B >$ is evaluated by the graphical equation of Fig. 5, where $C_i^{T(S)}$ are

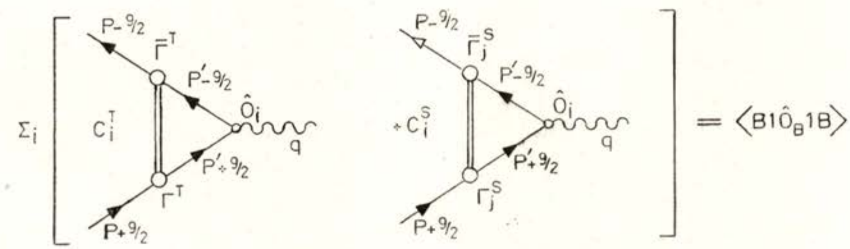


Fig. 5

products of SU(3) and SU(6) coefficients, and, apart from form factors which are supposed to be the same by SU(6), Γ^T and Γ_j^S are given by:

$$\Gamma^T = 1 \quad \text{and} \quad \Gamma_j^S = \frac{1}{\sqrt{3}} \gamma^5 \gamma_j$$

the factor $1/\sqrt{3}$ in Γ_j^s being a spin weighting factor. The coefficients C^T and C^S are easily determined by writing the SU(6) baryon wave function in a quark-diquark model ([42]). For the proton, for instance, we have

$$P \equiv \frac{1}{\sqrt{2}} \left[- \sqrt{\frac{2}{3}} S_1 \mathcal{N} + \sqrt{\frac{1}{3}} S_2 \mathcal{P} \right] + \frac{1}{\sqrt{2}} T_1 \mathcal{P} \qquad \text{(II. 47)}$$

where S_1, S_2 are sextet and, T_1 a triplet non-interacting diquark, and $\mathcal{N}, \mathcal{P}$ the interacting quark (*). Multiplying the square of these coefficients by 3, since each quark can interact, we get

$$C^T_{\mathcal{N}} = 0 , \qquad C^S_{\mathcal{N}} = 1 \tag{II. 48}$$

$$C^T_{\mathcal{P}} = \frac{3}{2} , \qquad C^S_{\mathcal{P}} = \frac{1}{2} .$$

For the neutron case $\mathcal{P}$ and $\mathcal{N}$ are interchanged in (II. 47) and (II. 48).

We evaluate the graphs of Fig 5 in the Breit frame of the baryon, and neglect all contributions from $\underset{\sim}{P}'$ in the numerator and from the time-like component of the $\tilde{S}$ diquark. In the case of the magnetic moment calculations the quark operator will be

$$\hat{O}^{\mu}_i \equiv e_i [F_1(q^2) \gamma^{\mu} - i K_i F_2(q^2) \sigma^{\mu\nu} q_{\nu}] \tag{II. 49}$$

where $F_1(q^2 = 0) = F_2(q^2 = 0) = 1$. The terms linear in q give the magnetic moment and both, γ^j and $\sigma^{j\nu}$ contribute. The quark anomalous magnetic moment gives a contribution $K_i \sigma^{j\nu} q_{\nu}$ that multiplies the normalization integral which we write schematically in the form

$$I_0 = \int H(P, P', q = 0) \, (P'_0 + M)^2 \, d^4 P' = 1 \tag{II. 50}$$

where form factors and denominators have been absorbed in $H(P, P', q)$ and $(P'_0 + M)^2$ comes from the interacting quark propagator numerators. The electric coupling γ^j gives two contributions, one coming from the q term in the quark propagators of the form $\frac{1}{2M} \sigma^{j\nu} q^{\nu}$ which contains as a factor the integral

$$I_1 = \int H(P, P', q = 0) \, (P'_0 + M) \, 2 M \, d^4 P' \tag{II. 51}$$

(*) $S_1 \equiv [\mathcal{P}\mathcal{P}]$, $S_2 \equiv \frac{1}{\sqrt{2}} [\mathcal{P}\mathcal{N} + \mathcal{N}\mathcal{P}]$, $T_1 \equiv \frac{1}{\sqrt{2}} [\mathcal{P}\mathcal{N} - \mathcal{N}\mathcal{P}]$.

and another form the q dependence of the nucleon spinors of the form $\frac{1}{2\,M_B}\,\sigma^{j\nu}\,q^{\nu}$ with a factor

$$I_2 = \int H(P, P', q=0)\,(P_0'^2 - M^2)\,d^4 P' \tag{II. 52}$$

From (II. 50), (II. 51), (II. 52) we have

$$I_0 = I_1 + I_2 = 1\,. \tag{II. 53}$$

Going further into a non-relativistic situation we assume now that the average value of $P_0'^2$ in I_2 equals that of M^2, which means that the interacting quark acts effectively as if on its mass shell. This assumption is consistent with using in (II. 49) only the on mass shell couplings. Neglecting integrals in P' and equating integrals in $P_0'^2$ to integrals in M^2 we reproduce the conditions for a non-relativistic model with quarks essentially at rest and static interactions. Equation (II. 52) then gives

$$I_2 = 0 \quad \text{and} \quad I_0 = I_1 = 1\,.$$

The equation of Fig. 5 gives then the non-relativistic SU(6)-quark model results for the magnetic moments, in the form,

$$\sum_{\nu=\mathcal{N},\mathcal{P},\lambda} \left\{ -\frac{1}{3} \sum_S C_\nu^S + \sum_T C_\nu^T \right\} e_\nu\,\mu_\nu = e\,\mu_B \tag{II. 54}$$

where the coefficients $-\frac{1}{3}$ and 1 arise from the commutation of the Γ_S^j and γ^j or $\sigma^{i\nu}$. We note that contributions from the integral I_2 or from timelike diquark components would have spoiled the additivity rule expressed by (II. 54). In particular for the proton we have,

$$\mu_{\text{proton}} \simeq \frac{1}{2\,M} + K_{\mathcal{P}}\,. \tag{II. 55}$$

The baryon axial vector weak couplings and pseudovector couplings to pseudoscalar mesons are similar to the magnetic couplings and the additivity results are derived in the same way.

In the present model the magnetic moment of the $\mathcal{N}$ and $\mathcal{A}$ quark differ from that of the $\mathcal{P}$ quark. We may write (II. 29) and (II. 30) in the form

$$K_{\mathcal{N}} = K_{\mathcal{P}}[1+\beta] \tag{II. 56}$$

and

$$K_{\lambda} = K_{\mathcal{P}}[1-\alpha] \tag{II. 57}$$

where β and α are, taking the experimental values for the vector meson masses and $\theta = 33°$, $\alpha = 0.40$ and $\beta = 0.07$. The factors α and β provide corrections to the usual non-relativistic calculations of magnetic moments. In the limit of quarks with only a anomalous magnetic moment the effect of the corrections α and β is shown in Table II. 5 ($\mu_{\text{proton}} = 2.793$).

TABLE II. 5

BARYON	Theoretical				EXPERIMENTAL
	No Correction $K_{\lambda} = K_{\mathcal{N}} = K_{\mathcal{P}}$		Correction α and β		
	μ/μ_{proton}	μ	μ/μ_{proton}	μ	
n	$-2/3$	-1.86	$-\frac{2}{3}\left(\frac{1+2/3\,\beta}{1+1/9\,\beta}\right)$	-1.93	-1.913
Λ	$-1/3$	-0.93	$-\frac{1}{3}\left(\frac{1-\alpha}{1+1/9\,\beta}\right)$	-0.57	-0.73 ± 0.16
Σ^+	1	2.793	$\frac{1-1/9\,\alpha}{1+1/9\,\beta}$	2.66	2.5 ± 0.5
Ω	-1	-2.793	$-\frac{1-\alpha}{1+1/9\,\beta}$	-1.71	?

In all cases the corrections act in the right direction and strongly reduce the magnetic moments of the Λ and Ω.

In this model other corrections could equally well have been considered, such as violations of SU(6) in the wave functions and contributions from I_2 integrals, and we have not a priori reason to neglect them compared to the α and β corrections.

5. Four body processes.

We now briefly review some of the attempts to evaluate the graph of Fig. 1c (Page 20) and its generalisations to n external particles. The principle of the evaluation is still the same as in graphs 1.a and 1.b: the underlying structure is provided by quarks. However in addition to the vertex functions one needs to incorporate, in the case of meson processes, the $2n$ quark amplitudes. With no dynamics explicitly included, the graph of Fig. 1c is a duality diagram of Harari and Rosner (43). The best realization of the physics suggested by duality diagrams is the Veneziano model (15) or, in general, a dual resonance model (44). Thus the most obvious way of evaluating the n particle amplitude T_n is by writing it in the form

$$T_n = \Gamma_{2n} \cdot B_{2n} \tag{II. 58}$$

where Γ_{2n} describes the vertex structures, i. e., the spin, SU(3), mass breaking factors at the connections of the n mesons to the $2n$ quarks, and B_{2n} is a dual resonance model function that describes the $2n$ quark interaction. Expressions in the form of Equation (II. 58) were used by different authors in particular Bardakci and Halpern (23), Mandelstam (24) and Delbourgo and Rotelli (45). The amplitude B_{2n} is the standard Veneziano $2n$ point function (46) for quarks. Projecting out the pole terms corresponding to the external lines the B_{2n} function can be reduced to the B_n function for mesons (24).

These models have the general properties of dual resonance models, namely they are crossing symmetric and duality constraints (Finite Energy Sum Rules) are exactly satisfied. They further have the advantage of allowing a rigorous inclusion of higher symmetries, in particular, SU(6). One could then hope to combine the successes of the Veneziano model with the successes of the SU(6) quark model classification of resonances. Several attempts were made [47] but there remains the problem of controlling the excessive degeneracy present in dual quark models.

In fact one of the difficulties of this sort of models is that they produce parity doublets in the meson trajectories, the trajectories are doubled (pairs of particles with the same usual quantum numbers), and there is mass degeneracy for each orbital excitation ($m_\pi = m_\rho$, for instance). The unwanted solutions can of course be simply neglected. The Carlitz and Kislinger [48] prescription to avoid parity doubing in fermion trajectories can be introduced using positive energy quark projection operators and fixed cuts in all channels. This ideia was developed by Bardakci and Halpern [49] and simplified later by Venturi [50]. In this process of refining the model, apart from the introduction of boson cuts without a clear physical meaning, the simplicity of the one (leading) term approximation is lost; cancellations between leading and non leading terms being required to achieve the eliminations of ghosts in the main trajectory.

The other important drawback of the dual relativistic quark model is its difficulty in achieving good low energy behaviour. Theoretically this corresponds to saying that soft pion limit theorems are not satisfied. This point however can be implemented by using vertex terms Γ in (II. 58) with convenient mass breaking factors. The required mass factors are basically the ones of Llewelyn Smith Model I as they appear in the vertex functions (II. 26) and (II. 27). In Refs. [23] and [24] the correct wave function for the ρ meson is used and these models are able to reproduce the usual $SU(6)_W$ results for coupling constants. In Refs. [24] and [45] with a wave function for the π meson different from our Equation (II. 26), $\Gamma_{\pi QQ} \propto \gamma^5 \cdot (1 + P/m_\pi)$ compared to $\Gamma_{\pi QQ} \propto \gamma^5 (1 + P/m_\rho)$, bad low energy behaviour is obtained. The improvement in the low energy behaviour with the relativistic quark model comes from the use of modified Veneziano amplitudes (free of parity doubling and with fixed cuts) and vertex structures as in Equation (II. 26) [51]. Extensions to this sort of work with applications to πN low energy scattering were recently made by Lebrun [52] and Venturi [53].

The main objection to these relativistic quark models lies probably in the fact that they are extremely complicated. Once one starts introducing extra terms to eliminate the weak points in the original versions of the model (too much degeneracy and bad low energy behaviour) it is difficult to see how to make it of practical use. Most of the effort so far has been directed to redeviring results already known, and previously obtained in a much more straightforward manner (low energy parameters, for instance).

In the next chapters we will try to combine the ideas of duality with more conventional versions of the quark model.

CHAPTER III

Duality: Quark model at high and low energy

1. Introduction.

The aim of this chapter and the next is to understand better the quak graph for a reaction $AB \rightarrow AB$. The limitations of a quark Feynman graph interpretation of these processes were explained in the previous chapter. The successes and complications met in other attempts were also briefly discussed there.

If we draw the graph of Fig. II 1c (page 20) without showing explicitly any quark interactions, i. e., with only quark lines and no vertex functions or bound states, it becomes obviously a planar quark duality diagram (45). Even without any dynamics in it, this diagram can be used as an important tool in studying strong interactions. It embodies the key idea of duality (54) that resonances in one dhannel are connected with exchange of particles in the crossed channel. Experimentally the existence or non-existence of a planar duality diagram has an observable consequence (55). When we are allowed to draw a planar diagram the total cross section, σ^T_{AB}, shows an appreciable energy dependence, bumps first, followed by a smooth falling curve as energy increases. When no quark diagram exists the cross-sections become roughly flat at very low energies (no bumps and no falling down curve). This connection between cross section behaviour and duality diagrams is fully obeyed experimentally.

At high energy, lab. energy $\simeq 30\,GeV$ in πN and KN processes, all cross sections tend to become constant (or slightly increasing with energy) regardless of the existence of a duality diagram. Thus at sufficiently high energy, probably all cross sections become similar in their dependence on the energy. We can then distinguish two components in the cross sections and are thus in this way led to the Freund-Harari conjecture (56): the cross section σ^T_{AB} can be described by two independent terms, one, given by the duality diagram $AB \rightarrow AB$ supplies the energy dependence, the other, Pomeranchuk-like term, gives the energy independent contribution.

The relation between duality diagrams and energy dependence of cross sections allows a Regge pole model interpretation in terms of degeneracy of exchanged particle trajectories. However the Regge pole model is too restrictive. A duality relation, valid for the imaginary part of the amplitudes, in that model imposes, as the phase is precisely defined, constraints in the real part of the amplitude. However the presence of strong cuts may spoil the Regge asymptotic phases and thus the relations involving the real parts of amplitudes derived from duality for the imaginary parts may become invalid. Failures of exchange degeneracy applied to differential cross sections in inelastic processes, for instance

$$K^- P \rightarrow \pi^- \Sigma^+ \quad \text{and} \quad \pi^+ P \rightarrow K^+ \Sigma^+$$

are well known (57). There are not such failures of the quark diagram tests of duality. For instance the cross section for $(K^+ p)$ and $(K^+ n)$ are roughly similar and show no important energy dependence at least up to $60\, GeV$, $\text{Im}\,(K^+ n \rightarrow K^0 p)$ seems to be zero (58) as expected from the non existence of a legal diagram. It would be interesting to check the prediction $\text{Im}\,(K^- p \rightarrow \Sigma^+ \pi^-) = 0$ because this is one of the reactions where Regge exchange degeneracy fails.

We will thus apply duality only to the imaginary part of the amplitude. The real part can in principle be determined as usual from dispersion relations. The quark model determines parameters required to describe the imaginary part of the amplitude, coupling constants at low energy and relations between cross sections at high energy. This information used in dispersion relations gives us the real part of the amplitudes.

We have seen in the last chapter how quark model in our relativistic version can be applied to determine coupling constants. We will discuss in Section III. 2 how quark model can be applied at high energy, basically using the idea of additivity (59, 60, 9). These low energy and high energy quark model calculations are apparently unconnected. However if duality works these calculations cannot be independent. This is what we show, first in a qualitative way, in Section III. 3, introducing the idea of additivity of quark-hadron duality diagrams. Quantitative predictions are given in Chapter IV.

Using information only from quark model calculations we develop a crude model for the imaginary part of the scattering amplitude in a reaction $A+B \rightarrow A+B$. The model simply states that at low energy we have the p wave resonances corresponding to the $L=0$ orbital excitations of the SU(6) quark model and the remainder is a continuous high energy curve to which the additivity relations apply. Such a model, supplemented with a factorization assumption in the high energy curve, is used to evaluate current algebra sum rules (Section III. 3) and dispersion relations (Section III. 4).

2. Additivity and duality.

In the high energy region, i. e., region where the cross sections become smooth functions of the energy, the additivity assumption as introduced in Refs. [59] and [60] states that for forward scattering the amplitude $T_{AB}(s, t \simeq 0)$ for AB scattering is simply given by the sum of all possible two body quark-quark and quark-antiquark contributions:

$$\operatorname{Im} T_{AB}(s, t) = \sum_{ij} \operatorname{Im} T_{ij}(s, t) \qquad \text{(III. 1)}$$

where T_{ij} is the amplitude for the scattering of quark i from A by quark j from B. Relation (III. 1) corresponds to the impulse approximation and is expected to be valid only near $t=0$, where multiple scattering can be neglected.

To introduce in the high energy quark model formalism the Freund-Harari conjecture we write

$$T_{ij} = T^{P}_{ij} + T^{E}_{ij} \delta_{i\bar{j}} \qquad \text{(III. 2)}$$

where T^P gives a flat contribution to the cross section and T^E gives the energy dependent part. By $\bar{j}$ we represent the antiparticle of quark j. In first approximation we assume the validity of SU(3): $T^P_{i\bar{j}} = T^P$, $T^E_{i\bar{j}} = T^E$. The energy independent part of the cross section for AB scattering is, from (III. 1) and (III. 2), proportional to the number of pairs of quark lines, i, j present in the process. This prescription gives immediately the famous ratios:

$$\lim_{s \to \infty} [\sigma^T(p\,p) : \sigma^T(p\,\pi) : \sigma^T(\pi\,\pi)] = 9 : 6 : 4 \qquad \text{(III. 3)}$$

The energy dependent part of the cross section exists only when there are quark-antiquark annihilations in the s channel.

The superscripts P and E in T^P and T^E are suggested from the language of Regge poles: P stands for Pomeranchuk and E for Exchange of particles. In our discussion we leave out Pomeranchuk type of contributions. If, when computing the energy dependent term instead of counting quark-antiquark annihilations we rearrange the $Q\bar{Q}$ pairs to form mesons and sum over these mesons our model with additivity becomes a particle exchange model. It is then not surprising that additivity with prescription (III. 2) can be made equivalent to an exchange model with Pomeran and particles being exchanged in t channel [61]. If we take meson-baryon (MB) scattering, an exchange model, say Regge Model, with Pomeran, vector and tensor mesons in t channel, with SU(3) and pure F coupling applied at the vertices gives the same relations between cross sections as quark model additivity. For example, in πN and KN scattering the use of equation (III. 2) provides the following relations:

$$f_{N f_0} = f_{N \omega} \qquad \text{(III. 4)}$$

$$f_{N A_2} = f_{N \rho} \qquad \text{(III. 5)}$$

$$f_{N \omega} = 3 f_{N \rho} \qquad \text{(III. 6)}$$

where $f_{N V(T)}$ represents the imaginary part of the (MB) amplitude corresponding to t channel exchanges with the quantum numbers of vector (tensor) meson $V(T)$. Equations (III. 4) and (III. 5) are the usual relations for exchange degeneracy of opposite parity vector and tensor mesons. Equation (III. 6), a usual quark model or SU(3) with pure F coupling relation, shows that exchange degeneracy does not occur for vector (or tensor) mesons of opposite G parity.

So far we have not used the concept of non exotic states, i. e., states which from the point of view of SU(3) belong to the $\underline{3}\times\overline{\underline{3}}$ (Mesons) and $\underline{3}\times\underline{3}\times\underline{3}$ (Baryons) representations (Equations (I. 1) and (I. 2)). In connection with duality, in particular with the idea that exotics in the s channel correspond to exchange degeneracy in t channel, the above classification can be sucessfully applied in scattering of mesons by other mesons or baryons. However for $B\overline{B}$ scattering the connection between s channel resonances and t channel exchanges certainly needs to be reinterpreted [62]. At the same time one may wonder about the validity of the exchange part of Equation (III. 2) in $B\overline{B}$ scattering when simultaneous $Q\overline{Q}$ annihilations can take place [63,64]. Because of these difficulties from now on we only consider scattering processes with at least one of the particles being a meson.

Now we look at additivity from a slightly modified point of view. The mesons are seen as some elementary quanta of a field used to investigate the quark structure of hadrons. We thus apply additivity by counting basic (MQ) amplitudes. From the πN and KN additivity (exchange model) relations without Pomeron term we derive for the MQ amplitudes $(Q\equiv\mathcal{N},\mathcal{P},\lambda)$ expressions like:

$$3\,(K^-\mathcal{P})=\frac{1}{2}\,(f_{Nf_0}+f_{N\omega})+\frac{3}{2}\,(f_{N\rho}+f_{NA_2}) \qquad \text{(III. 7)}$$

$$3\,(K^+\mathcal{P})=\frac{1}{2}\,(f_{Nf_0}-f_{N\omega})+\frac{3}{2}\,(-f_{N\rho}+{}_{NA_2}) \qquad \text{(III. 8)}$$

$$3\,(K^+\mathcal{N})=\frac{1}{2}\,(f_{Nf_0}-f_{N\omega})+\frac{3}{2}\,(f_{N\rho}-f_{NA_2}) \qquad \text{(III. 9)}$$

$$3\,(K^-\mathcal{N})=\frac{1}{2}\,(f_{Nf_0}+f_{N\omega})-\frac{3}{2}\,(f_{N\rho}+f_{NA_2}) \qquad \text{(III. 10)}$$

$$3\,(\pi^+\mathcal{P})=f_{Nf_0}-3\,f_{N\rho} \qquad \text{(III. 11)}$$

$$3\,(\pi^-\mathcal{P})=f_{Nf_0}+3\,f_{N\rho} \qquad \text{(III. 12)}$$

We notice that Equations (III. 7) to (III. 12) are written in a form that allows them to be interpreted in the sense of Finite Energy Sum Rules [65] (FESR), i. e., in both sides we have the same quantity but in the left hand side written as an s channel process and in the right hand side as a t channel exchange. What we show next is that Equations (III. 7) to (III. 12) are duality consistent, resonances and particle exchanges being interrelated. To see that we remark first that once we treat basic quark interactions with mesons we are effectively considering the quark representation states as non exotic extending in this way the field of the allowed representations: $Q\overline{Q}$, QQQ and Q. From SU(3) the fictitious process $MQ \to MQ$ has the decomposition:

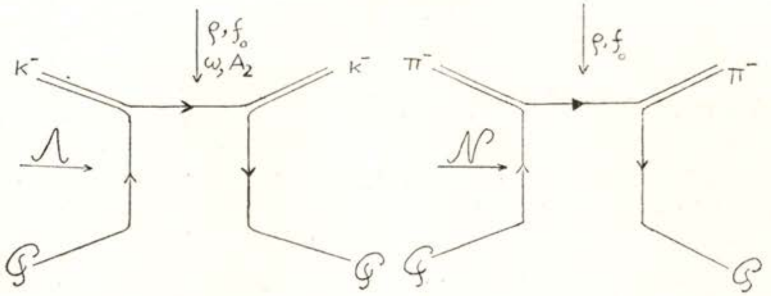
$$\underline{8} \times \underline{3} = \underline{3} + \underline{15} + \overline{6}$$

and the only possible resonances must belong to the quark representation, i. e., must have one of the following sets of quantum members:

$$I = 1/2, \quad I_Z = \pm 1/2, \quad Y = 1/3, \quad S = 0$$
$$I = 0, \quad I_Z = 0, \quad Y = -2/3, \quad S = -1.$$

We can now analyse Equations (III. 7) to (III. 12) as FESR's. The left hand sides of relations (III. 7) and (III. 12) allow s channel resonances, the right-hand sides show a non vanishing contribution. The left hand sides of (III. 8) and (III. 9) are exotic: $(K^+ \mathcal{P})$ and $(K^+ \mathcal{N})$ have $S = +1$. The right hand sides show vanishing contributions from the exchange degeneracy relations (III. 4) and (III. 5). Note that the isospin equivalent processes at nucleon level, $(K^+ p)$ and $(K^+ n)$, are also exotic and exchange degeneracy also makes the right hand side of the corresponding sum rule vanish. The left hand sides of Equations (III. 10) and (III. 11) are exotic, $(K^- \mathcal{N})$ has $I = 1$ and $(\pi^+ \mathcal{P})$ has $I = 3/2$. The right hand sides because of exchange degeneracy and relation (III. 6) also vanish. At nucleon level, $(K^- n)$ and $(\pi^+ p)$ processes, the s channel is not exotic (Σ resonances in one case and Δ resonances in the other), and the exchange contributions are also non-vanishinh. The extra exchange degeneracy that occurs at quark level because of the equality of the quark couplings to isoscalar and isovector particles is reflected back in the absence of resonances in the $I=1$ and $I=3/2$ channel.

These results can be visualized if one thinks in terms of (s,t) duality diagrams, the ones that refer to forward scattering and can thus be related to additivity. We are allowed to draw



a

b

Fig. 1

them only for the processes where s channel resonances and t channel particle exchanges occur, $(K^- \mathscr{P})$ and $(\pi^- \mathscr{P})$, Equations (III. 7) and (III. 12). These are shown in Figs. (1. a, b).

If we now come back to MB and MM scattering we immediately note that the MB (or MM) (s,t) duality diagram can

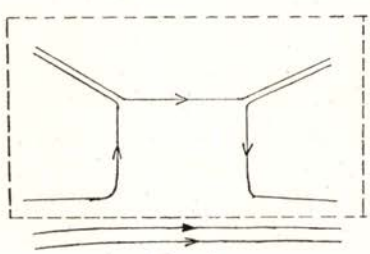


Fig. 2

be obtained by summing MQ duality graphs in the presence of quark lines (Fig. 2).

The important thing Fig. 2 is telling us is that additivity can be interpreted as additivity of MQ duality diagrams. Being primarily a concept valid at high energy additivity can thus be extended to the low energy region provided we find a convenient

F.E.S.R. that allows the comparison of high energy to low energy parameters. The problem of extracting quantitative predictions from the suggestive Fig. 2 is discussed in chapter IV. We need first to develop a model to evaluate the imaginary part of MM and MB scattering amplitudes. At te same time a deeper understanding of dual resonance models is required before turning back to that problem.

3. Current Algebra Sum Rules.

We introduce now a model to evaluate the imaginary part of the amplitude T^E_{AB} in the forward direction. It simply states:

(*i*) at low energy $\mathrm{Im}\, T$ is given by contributions from the SU(6) quark model $L=0$ states (first p wave resonances)

(*ii*) at high energy, i. e., above the first resonances, $\mathrm{Im}\, T$ satisfies the quark model additivity relations.

This is, of course, a very crude model but it has the advantage of incorporating the basic information given by quark model. The coupling constants required from (*i*) are evaluated using, for instance, the techniques developed in chapter II. The energy dependence of $\mathrm{Im}\, T$ at high energy is not determined in quark model (contrary to Regge model that predicts $\mathrm{Im}\, T \sim s^{\alpha(t)-1}$) and we will extrapolate, when required, high energy experimental fits. As the model is constructed only for the resonance-exchange part of the amplitude when using (*ii*) to evaluate, for example, dispersion integrals, we must subtract the Pomeron exchange term.

We have already referred to the equivalence of using quark model additivity or an exchange model at high energy. When comparing two processes, πN to ρN, say, it is then natural to introduce a third assumption in our model:

(*iii*) at high energy the imaginary parts of the amplitudes for different processes are related by factorization.

With our model for $\mathrm{Im}\, T$, using (*i*), (*ii*) and (*iii*) we evaluate some Current Algebra Sum Rules, first for quark reactions and after for nucleon reactions.

To generate Current Algebra Sum Rules there exists a general procedure: [66] sandwich the commutator of two operators (local or integrated operators) between two states, insert intermediate states and separate Born terms (i. e. one particle intermediate state), from physical region contributions. In general the sum rules have the structure:

$$(\text{Isospin factor}) = (\text{Born}) + (\text{Continuum})$$

and the (Continuum) is an integral containing the imaginary part of an amplitude. This term can also be considered as the dispersion integral for the amplitude evaluated at $\nu \equiv \left(\frac{s-u}{4M}\right) = \nu_{\text{Born}} = 0$.

To start we write the Adler-Weisberger [67, 68] sum rule for $\pi\mathcal{Q}$ scattering in the standard form given above:

$$1 = g^2_{AQ} + F^2_\pi \, G(0) \tag{III. 13}$$

with

$$G(0) = \frac{1}{\pi} \int_{\nu_{\text{th}}}^{\infty} \frac{d\nu}{\nu} [\sigma_- - \sigma_+] \tag{III. 14}$$

where $F_\pi = 0.85\, m_\pi$ is the π annihilation parameter as given by the Goldberger-Treiman relation and $[\sigma_- - \sigma_+]$ is the difference of the $(\pi^- \mathscr{P})$, $(\pi^+ \mathscr{P})$ total cross sections. The low energy contribution to the sum rule, contribution type (*i*), is explicitly shown in the Born term amplitude g^2_{AQ}. For the high energy contribution, type (*ii*), we use additivity,

$$\sigma(\pi^- \mathscr{P}) - \sigma(\pi^+ \mathscr{P}) = \sigma(\pi^- p) - \sigma(\pi^+ p) \tag{III. 15}$$

and, to evaluate (III. 14), an experimental fit to πN scattering [69] (units of m_π):

$$[\sigma_- - \sigma_+] = 1.70\, \nu^{-0.7} \tag{III. 16}$$

For simplicity we do not consider possible Pomeranchuk theorem violating terms. In strongly convergent sum rules, as the Adler--Weisberger relation, these terms are not important [70]. From Equations (III. 16), (III. 14) and (III. 13) we obtain $g_{AQ} = 0.67$ to be compared to the additivity value $g_{AQ} \simeq 0.707$. We interpret these numbers as meaning that the model is not unreasonable and adjust $G(0)$ to reproduce the additivity result for g_{AQ}:

$$G(0) = \frac{1}{2} \frac{1}{F_\pi^2} \tag{III. 17}$$

We consider next a sum rule for quark Compton scattering, the Cabibbo-Radicati sum rule (71). Using again additivity — the photon has a $\bar{Q}Q$ structure in Quark-Model — we write the sum rule in the form:

$$0 = -K_1'^V(0) + \frac{1}{2}(K_2^V(0))^2 + \frac{1}{f_\rho^2} G(0) \tag{III. 18}$$

where K_1^V and K_2^V are the Dirac and Pauli electromagnetic form factors,

$$K_1'^V(0) = \frac{\partial}{\partial t} K_1^V(t)|_{t=0} \simeq 1/m_\rho^2 \text{ and } K_2^V(0) \simeq 1/m_\rho.$$

From (III. 17) and (III. 18) we derive the KSFR relation (38);

$$f_\rho F_\pi = m_\rho. \tag{III. 19}$$

Similarly for the neutral pion photoproduction sum rules of Fubini, Furlan, Rosseti (72) we have

$$0 = -\frac{1}{2} K^V + \frac{M}{g_Q} A_1^{(+)}(0) \tag{III. 20}$$

$$0 = -\frac{1}{2} K^S + \frac{M}{g_Q} A_1^{(0)}(0) \tag{III. 21}$$

were $A_1^{(+)}$ and $A_1^{(0)}$ are CGLN (73) amplitudes. Writing dispersion relations for $A_1^{(+)}(0)$ and $A_1^{(0)}(0)$ and considering again only the Regge type of contributions, the $A_1^{(+)}$ amplitude will be dominated by ω exchange and $A_1^{(0)}$ by ρ exchange. Quark model additivity immediately gives for the ratio (III. 20) to (III. 21)

$$f_\rho^{-1} / f_\omega^{-1} = -3 \tag{III. 22}$$

the usual SU(3) relation. Relating $A_1^{(+)}(0)$ to $G(0)$ by factorization and assuming that the ratio of the residues of off mass shell particles is the same as when they are on the mass shell we derive [74],

$$A_1^{(+)}(0) = \frac{1}{2 f_\rho^2} m_\rho^2 \, g_{\rho\omega\pi} \, K^V G(0). \qquad \text{(III. 23)}$$

Combining (III. 23), (III. 20), (III. 17) and (III. 19) we obtain a well known $SU(6)_W$ result [40]:

$$g_{\rho\omega\pi} = \frac{2}{m_\rho} f_\rho. \qquad \text{(III. 24)}$$

These SU(6) type of results, Equations (III. 24), (III. 22) and (III 19) come from simultaneous manipulations, through our model, of low energy and high energy quark model calculations. This overall SU(6) consistent picture can only be understood in terms of duality as will be shown in chapter IV.

With our model, i. e., as specified in conditions (*i*), (*ii*) and (*iii*), we evaluate the same set of sum rules — Adler-Weisberger, Cabibbo-Radicati, Fubini et al. sum rules — for nucleon targets. The results are summarized in Table III. 1.

TABLE III. 1

Current Algebra Sum Rules for nucleon target
Evaluation using prescription (*i*), (*ii*) and (*iii*)

Sum Rule		L. H. S.	R. H. S.				References to detailed evaluations
			Born term	Δ (1236)	High Energy	Total	
Adler-Weisberger		1	$+1.50$	-1.06	$+0.5$	0.94	[67, 68]
Cabibbi-Radicati		0	$+0.140/m_\rho^2$	$-0.819/m_\rho^2$	$+0.5/m_\rho^2$	$-0.179/m_\rho^2$	[71, 75]
Fubini et al.	$A_1^{(+)}$	0	$-1.85/(2m_N)$	$+1.55/(2m_N)$	~ 0	$-0.30/(2m_N)$	[72, 76]
	$A_1^{(0)}$	0	$+0.06/(2m_N)$	0	$-0.7/(2m_N)$	$-0.54/(2m_N)$	

Except for the last sum rule the terms in the right hand side tend to compensate among themselves and reproduce the left hand side value. The agreement is particularly good for the Adler-Weisberger relation. In the photoproduction sum rule for isoscalar photons the high energy term is too large. This is not a surprise because in photoproduction we also have exchanges of unnatural parity particles (B meson) and our model only takes into account natural parity exchanges. A break of the factorization assumption in photoproduction may also explain the bad agreement in this case.

4. Low energy parameters.

In this section we use our ansatz for the imaginary part of the amplitude to compare current algebra predictions and dispersion relation calculations of low energy parameters. We first establish a bridge between dispersion relations and current algebra showing that combining the usual dispersion relations with an expansion of the amplitude in the variable ν we simulate, working on mass shell, current algebra results: the Adler-Weisberger relation [67, 68] and the Adler self-consistent condition [77].

Let us consider the helicity non flip amplitude at $t=0$ for πN scattering $\left(\text{in general } \pi T \text{ scattering, } T \text{ with spin } \frac{1}{2}\right)$,

$$A'(\nu, t=0) = A(\nu, 0) + \nu B(\nu, 0) \qquad \text{(III. 25)}$$

where A and B are the usual invariant amplitudes [78]. We assume that $A'(\nu, 0)$ can be expanded around $\nu=0$ in a power series in ν. The scale is set up by the target mass m_N, i. e., the expansion is valid for $\nu/m_N << 1$. As we are interested in the amplitude at threshold, $(\nu = m_\pi) > (|\nu_B| = m_\pi^2/2\, m_N)$ the expansion cannot be valid for Born terms. Thus before expanding we separate out the Born term, B_b, and apply the expansion in ν only for the non pole part of the amplitude, $\bar{A'}$

$$A'(\nu, t) = \bar{A'}(\nu, t) + \nu B_b(\nu, t) \qquad \text{(III. 26)}$$

with

$$\bar{A}'(\nu, t) = A(\nu, t) + \nu \bar{B}(\nu, t). \tag{III. 27}$$

We expand now (III. 27):

$$\bar{A}'(\nu, 0) = \bar{A}'(0) + \left.\frac{\partial \bar{A}'(\nu)}{\partial \nu}\right|_{\nu=0} \nu + \frac{1}{2} \left.\frac{\partial^2 \bar{A}'(\nu)}{\partial \nu^2}\right|_{\nu=0} \nu^2 + \cdots. \tag{III. 28}$$

It is convenient to treat separately the s, u crossing even, $A'^{(+)}$, and crossing odd $A'^{(-)}$ amplitudes. For the $A'^{(+)}$ amplitude we have:

$$\bar{A}'^{(+)}(\nu) = A^{(+)}(0) + \left[\frac{\partial \bar{B}^{(+)}}{\partial \nu} + \frac{1}{2} \frac{\partial^2 A^{(+)}}{\partial \nu^2}\right]_{\nu=0} \nu^2 + O(\nu^4) \tag{III. 29}$$

and, in particular for $\nu = m_\pi$

$$4\pi\left(1 + \frac{m_\pi}{m_N}\right) a_0^{(+)} = A^{(+)}(0) + m_\pi B_b^{(+)}(m_\pi) + \\ + \left[\frac{\partial \bar{B}^{(+)}}{\partial \nu} + \frac{1}{2} \frac{\partial^2 A^{(+)}}{\partial \nu^2}\right]_{\nu=} m_\pi \tag{III. 30}$$

where a_0 is the s wave scattering length.

For then $A'^{(-)}$ amplitude we have:

$$\bar{A}'^{(-)}(\nu) = \left[\bar{B}^{(-)} + \frac{\partial A^{(-)}}{\partial \nu}\right]_{\nu=0} \nu + O(\nu^3) \tag{III. 31}$$

and

$$4\pi\left(1 + \frac{m_\pi}{m_N}\right) a_0^{(-)} = B_b^{(-)}(m_\pi)\, m_\pi + \left[\bar{B}^{(-)} + \frac{\partial A^{(-)}}{\partial \nu}\right]_{\nu=0} m_\pi \tag{III. 32}$$

Now we use the dispersion relations for the A and B amplitudes to evaluate the right hand sides of (III. 30) and (III. 32). Following Hamilton and Woolcock (79) we make a subtraction only for the $A^{(+)}$ amplitude. One then has:

$$\operatorname{Re} A^{(+)}(\nu) = A^{(+)}(0) + \frac{2}{\pi}\nu^2 \int_{m_\pi}^{\infty} \frac{d\nu'}{\nu'^2}\nu' \frac{\operatorname{Im} A^{(+)}(\nu')}{\nu'^2 - \nu^2} \tag{III.33}$$

$$\operatorname{Re} \overline{B}^{(+)}(\nu) = \frac{2}{\pi}\int_{m_\pi}^{\infty} d\nu' \, \nu \frac{\operatorname{Im} B^{(+)}(\nu')}{\nu'^2 - \nu^2} \tag{III.34}$$

$$\operatorname{Re} A^{(-)}(\nu) = \frac{2}{\pi}\int_{m_\pi}^{\infty} d\nu' \, \nu \frac{\operatorname{Im} A^{(-)}(\nu')}{\nu'^2 - \nu^2} \tag{III.35}$$

$$\operatorname{Re} \overline{B}^{(-)}(\nu) = \frac{2}{\pi}\int_{m_\pi}^{\infty} d\nu' \, \nu' \frac{\operatorname{Im} B^{(-)}(\nu')}{\nu'^2 - \nu^2} \tag{III.36}$$

and

$$B_b^{(+)}(\nu) = (4\pi)\frac{4 m_N}{m_\pi^2} f^2 \frac{\nu}{\nu_B^2 - \nu^2} \tag{III.37}$$

$$B_b^{(-)}(\nu) = (4\pi)\frac{4 m_N}{m_\pi^2} f^2 \frac{\nu_B}{\nu_B^2 - \nu^2}. \tag{III.38}$$

From Equations (III.33) to (III.36) one immediately obtains:

$$\frac{1}{2}\frac{\partial^2 A^{(+)}}{\partial \nu^2}\bigg|_{\nu=0} = \frac{2}{\pi}\int_{m_\pi}^{\infty} \frac{d\nu'}{\nu'^3} \operatorname{Im} A^{(+)}(\nu') \tag{III.39}$$

$$\frac{\partial \overline{B}^{(+)}}{\partial \nu}\bigg|_{\nu=0} = \frac{2}{\pi}\int_{m_\pi}^{\infty} \frac{d\nu'}{\nu'^2} \operatorname{Im} B^{(+)}(\nu') \tag{III.40}$$

$$\frac{\partial A^{(-)}}{\partial \nu}\bigg|_{\nu=0} = \frac{2}{\pi}\int_{m_\pi}^{\infty} \frac{d\nu'}{\nu'^2} \operatorname{Im} A^{(-)}(\nu') \tag{III.41}$$

$$\operatorname{Re} \overline{B}^{(-)}(0) = \frac{2}{\pi}\int_{m_\pi}^{\infty} \frac{d\nu'}{\nu'} \operatorname{Im} B^{(-)}(\nu'). \tag{III.42}$$

Neglecting at $\nu^2 = m_\pi^2$, ν_B^2 in the denominators of the Born terms, and using Equations (III.37) to (III.42) in (III.29) and (III.31) we obtain:

$$4\pi\left(1+\frac{m_\pi}{m_N}\right)a_0^{(-)}=(4\pi)\frac{2}{m_\pi}f^2+ \qquad \text{(III. 43)}$$

$$+\frac{2}{\pi}m_\pi\int_{m_\pi}^{\infty}\frac{d\nu'}{\nu'^2}\,\mathrm{Im}\,A'^{(-)}(\nu')\,d\nu'$$

$$4\pi\left(1+\frac{m_\pi}{m_N}\right)a_0^{(+)}=A^{(+)}(0)-(4\pi)\frac{4\,m_N}{m_\pi^2}f^2+ \qquad \text{(III. 44)}$$

$$+\frac{2}{\pi}m_\pi^2\int\frac{d\nu'}{\nu'^3}\,\mathrm{Im}\,A'^{(+)}(\nu')\,d\nu' .$$

Equation (III. 43) is formally identical to the Adler-Weisberger relation (67, 68) with Weinberg's expression for the scattering lengths (80). Equation (III. 44), we will see, leads to a on shell Adler condition (77). Equations (III. 43) and (III. 44) are first order approximations to the dispersion relations for $A'^{(-)}$ and $A'^{(+)}$ and the connection between these relations and current algebra results was already noticed in Ref. [81].

We thus take the point of view that the current algebra approximation is equivalent to a on shell treatment with an expansion of the amplitudes around the point $\nu \simeq 0$, $t \simeq 0$. To see how good the approximation is we have plotted in Graph 3 $\bar{A}'^{(\pm)}$ against (ν/m_π) using the recent Nielsen's tables (82) for amplitudes in the unphysical region. With our expansions in ν we require a linear dependence on ν for $\bar{A}'^{(-)}$ [Equation (III. 31)], and a parabolic one for $\bar{A}'^{(+)}$ [Equation (III. 29)]. The straight line

$$\bar{A}'^{(-)}(\nu)=-0.66\,\nu \qquad \text{(III. 45)}$$

and the parabola

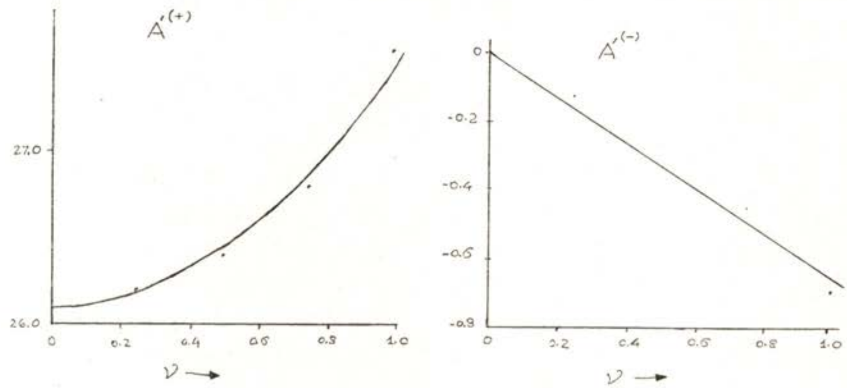
$$\bar{A}'^{(+)}(\nu)=26.1+1.4\,\nu^2 \qquad \text{(III. 46)}$$

fit reasonably well the tabulated points.

To determine low energy parameters, s and p waves scattering lengths, we need to evaluate the amplitudes at threshold. For the s wave scattering lengths we have Equations (III. 43) and (III. 44) and for p waves the relation

$$\frac{1}{2\,m_N}\,\frac{1}{4\pi}\,\mathrm{Re}\,[m_\pi\,B^I(m_\pi,0)]=(a_{1-}^I-a_{1+}^I)+\frac{m_\pi}{4\,m_N^2}\,a_0^I \qquad \text{(III. 47)}$$

where I stands for isospin, $a_{1\pm}$ are the p wave scattering lengths, $l=1$, $j=l\pm 1/2$. For the calculations we use Equations (III. 37) to (III. 42). The left hand sides (L. H. S.) and subtraction constants were evaluated from the fits to Nielsen's tables [Fig. 3 and similar curves to the $\bar{B}$ amplitudes]. The first p wave resonances, $N(938)$ — Born term — and $\Delta(1236)$ were expli-



$A'^{(+)}(\nu)$ and $A'^{(-)}(\nu)$ in the unphysical region
$t=0$, $0 \leqslant \nu \leqslant 1$ $(m_\pi = 1)$

Fig. 3

citly evaluated. For the high energy part of the (—) amplitude we used our ansatz with Equation (III. 17) and factorization, i. e.,

$$\operatorname{Im} A'(\nu) : \operatorname{Im}(\nu B(\nu)) : \operatorname{Im} A(\nu) = 1 : (1 + k^V) : (-k^V) \qquad \text{(III. 48)}$$

In Table III. 2 we show the results of our calculations and, for comparison, the high energy contribution required to satisfy exactly the equations, i. e., to have L. H. S. = R. H. S.

We note that for all the (+) amplitudes almost complete saturation occurs with the inclusion of the N and $\Delta(1226)$. A philosophy of resonance saturation for the $A^{(+)}$, $B^{(+)}$ and $A'^{(+)}$ amplitudes is then adequate and there is no need of knowing the high energy behaviour to estimate low energy parameters. In particular from the $A'^{(+)}$ amplitude we have approximately

$$A^{(+)}(0) \simeq \frac{g^2}{m_N}$$

an on shell version of the Adler condition.

TABLE III.2

Amplitude		Equation	L. H. S.	R. H. S.			High Energy Contribution	
				Constant	Born term	$\Delta(1236)$	required to have R. H. S. = L. H. S.	Predicted
$\frac{1}{4\pi}A$	(+)	III. 33	$\left(\frac{2m_N}{m_\pi^2}\right)0.190$	$\left(\frac{2m_N}{m_\pi^2}\right)0.156$	/	$\frac{2m_N}{m_\pi^2}(0.0377)$	$-\left(\frac{2m_N}{m_\pi^2}\right)0.004$	/
	(−)	III. 35	$-\left(\frac{1}{m_\pi}\right)0.732$	/	/	$-\left(\frac{1}{m_\pi}\right)0.588$	$-\left(\frac{1}{m_\pi}\right)0.144$	$-\left(\frac{1}{m_\pi}\right)0.137$
$\frac{m_\pi}{4\pi}B$	(+)	III. 34	$-\left(\frac{2m_N}{m_\pi^2}\right)0.191$	/	$-\left(\frac{2m_N}{m_\pi^2}\right)0.162$	$-\left(\frac{2m_N}{m_\pi^2}\right)0.0317$	$\left(\frac{2m_N}{m_\pi^2}\right)0.003$	/
	(−)	III. 36	$\left(\frac{1}{m_\pi}\right)0.840$	/	$\left(\frac{1}{m_\pi}\right)0.162$	$\left(\frac{1}{m_\pi}\right)0.495$	$\left(\frac{1}{m_\pi}\right)0.183$	$\left(\frac{1}{m_\pi}\right)0.174$
$\frac{1}{4\pi}A'$	(+)	III. 44	$-\left(\frac{2m_N}{m_\pi^2}\right)0.001$	$\left(\frac{2m_N}{m_\pi^2}\right)0.156$	$-\left(\frac{2m_N}{m_\pi^2}\right)0.162$	$\left(\frac{2m_N}{m_\pi^2}\right)0.0060$	$\left(\frac{2m_N}{m_\pi^2}\right)0.001$	/
	(−)	III. 43	$\left(\frac{1}{m_\pi}\right)0.108$	/	$\left(\frac{1}{m_\pi}\right)0.162$	$-\left(\frac{1}{m_\pi}\right)0.093$	$\left(\frac{1}{m_\pi}\right)0.039$	$\left(\frac{1}{m_\pi}\right)0.037$

For the (—) amplitudes in all cases the high energy curve gives an important contribution. The quantities predicted by our ansatz agree well with the quantities required to satisfy exactly the equations. The agreement for the $A'^{(-)}$ amplitude is not surprising because the corresponding equation is the Adler-Weisberger relation which we have checked before. More interesting is the check of factorization for the imaginary part of the amplitudes (Equation (III. 48). Our model gives for the ratios of the high energy contributions,

$$A'^{(-)} : \nu B^{(-)} : A^{(-)} = 1 : 4.7 : (-3.7)$$

in excellent agreement with the ratios obtained from the high energy contributions required to exactly satisfy Equations (III. 35), (III. 36) and (III. 43):

$$= 1 : 4.69 : (-3.7) .$$

It is now obvious that our simple ansatz can give correct predictions of s and p wave scattering lengths in πN scattering [using Equations (III. 43), (III. 44) and (III. 47) and taking the various contributions from Table III. 2]. A more detailed discussion of the πN case is contained in Ref. [21]. Using p wave resonance saturation for the (+) amplitudes and applying quark model additivity and factorization for the high energy part of the (—) amplitudes we can evaluate low energy parameters in other meson-baryon scattering processes. For the s wave scattering length, having in mind the Adler condition, the values obtained are close to the Weinberg's formula, as we will see in chapter IV. In Table III. 3 we show the predicted values for the p wave scattering length combination, $(a_{1-}^{(-)} - a_{1+}^{(-)})$, $I_t = 1$ in $\pi N, \pi \Sigma$ and $\pi \Xi$ scattering. The $I_t = 0$ scattering lengths are simply given in our model by the first p wave resonance contributions.

For completeness we include in Table III. 4 the set of low energy parameters, s and p wave scattering lengths and s wave effective ranges, for the hypothetical πQ scattering. They were obtained in Ref. [21] using dispersion relations. The high energy contributions were evaluated using additivity and experimental fits to πN scattering.

TABLE III. 3

$(a_{1-}^{(-)} - a_{1+}^{(-)})$ p-wave scattering lengths

Process	Prediction	Experiment
πN	$+0.061$	$+0.057$
$\pi \Sigma$	$+0.000$	?
$\pi \Xi$	-0.024	?

TABLE III. 4

Low Energy Parameters in πQ scattering (*)
$(m_\pi = 1\,,\ f_Q^2 \simeq 0.029)$

			Isospin Combination (+)	Isospin Combination (−)
s wave	Scatt. Lengths		~ 0	$+4\, f_Q^2$
s wave	Effective Range		$+2.24\, f_Q^2$	$+f_Q^2$
p wave	Scatt. Lengths	$l+1/2$	$+\frac{2}{3} f_Q^2$	$-0.79\, f_Q^2$
p wave	Scatt. Lengths	$l-1/2$	$-\frac{4}{3} f_Q^2$	$-0.38\, f_Q^2$

(*) f_Q is the $QQ\pi$ pseudovector coupling constant: $f_Q^2 = \frac{g_Q^2}{4\pi}\left(\frac{m_\pi}{2\,M_Q}\right)^2$

Before ending this chapter let us comment on the current algebra techniques to predict low energy parameters ([80, 83, 84]). In the *PCAC* treatment one expresses the off mass shell amplitude as a sum of two terms, one, direct term, corresponds to the scattering of axial vector currents by an on mass shell target, the other is the current algebra commutator term. Discussing the various ways in which this scheme can be applied, in particular the difficulties related to mass extrapolations, Huang and Urani ([85]) arrived to the conclusion that the scheme is in fact equivalent to a full on the mass shell treatment with s channel resonances *and* a ρ exchange term.

Comparing the current algebra predictions on low energy parameters in πN scattering to the rigorous calculations based on dispersion relations ([79, 86]) one observes that they roughly agree when *either* one or other of the above terms is dominant but discrepancies occur when both give important contributions. For instance the commutator alone (or a ρ exchange term alone) ([87]) gives a good prediction in the determination of the «universal» $a_0^{(-)}$ scattering length,

$$a_0^{(-)} \simeq \frac{1}{4\pi}\left(\frac{f_\rho}{m_\rho}\right)^2 . \qquad \text{(III. 49)}$$

Note that Equation (III. 43) with our ansatz also gives this value. Working for simplicity in the πQ case, the Born term alone gives

$$\frac{1}{4\pi}\;\frac{1}{2}\,(g_Q/M)^2 \simeq \frac{1}{4\pi}\;\frac{1}{2}\left(\frac{f_\rho}{m_\rho}\right)^2$$

and the high energy part, Equation (III. 17), gives the same quantity. Putting the two contributions together we recover (III. 49). Another good current algebra prediction is in the $a_{1\pm}^{(+)}\, p$ wave scattering lengths where direct term saturation alone gives a good result. This is related to the fact that dispersion relations for $(+)$ amplitudes are, as we have seen, well satisfied with a few s channel resonances (N and $\Delta(1236)$).

However current algebra predictions are in strong disagreement with dispersion relations in the case of the $a_{1-}^{(-)}\, p$ wave scattering length where both direct and ρ exchange term contribute appreciably. The reason for this disagreement is that effectively double counting occurs adding s and t channel terms. To determine the $a_{1\pm}^{(-)} p$ wave scattering lengts we need Equation (III. 36) for the real part of the B amplitude. In the current algebra treatment of the ρ exchange term factorization is applied to the real part of the amplitude, i. e.,

$$\mathrm{Re}\,[m_\pi B^{(-)}(m_\pi, 0)]_\rho = (1 + k^V)\,[\mathrm{Re}\,A'^{(-)}(m_\pi, 0)]_\rho \tag{III. 50}$$

and, from (III. 49),

$$= (1 + k^V)\left(\frac{f_\rho}{m_\rho}\right)^2. \tag{III. 51}$$

In our model the current algebra direct term corresponds to the low energy contributions in the dispersion integrals (N, Δ), the ρ exchange term has the counterpart in the high energy curve. As we apply factorization only to the imaginary part of the amplitude,

$$\mathrm{Im}\,[\nu B^{(-)}(\nu, 0)] = (1 + k^V)\,\mathrm{Im}\,A'^{(-)}(\nu, 0) \tag{III. 52}$$

we obtain a contribution to $\mathrm{Re}\,B^{(-)}(m_\pi, 0)$ given by,

$$\int \frac{\mathrm{Im}\,[\nu B^{(-)}(\nu, 0)]}{\nu^2}\, d\nu = (1 + k^V)\, G(0) \tag{III. 53}$$

and from (III. 17) and the KSFR relation,

$$= \frac{1}{2}(1 + k^V)\left(\frac{f_\rho}{m_\rho}\right)^2 \tag{III. 54}$$

which is half of the quantity given by the exchange model for the real part of the amplitude. As we have shown, with our factorization prescription, we reproduce fairly well the high energy contribution to the dispersion integrals. Current algebra calculations over estimate the exchange term: a full exchange term cannot be taken at the same time with direct channel terms. The exchange type term represents t channel contributions and it should be used to describe only the region above the s channel resonances. It is interesting to remark that in the current algebra treatment as the number of resonances included in the direct term increases — thus apparently improving the accuracy of the calculation — the prediction for the $a_{1-}^{(-)}$ scattering length become worse [88].

Our conclusion is that in the way current algebra is applied to the determination of low energy parameters it violates the basic idea of duality that at a given energy only either an s channel or a t channel picture of the process should be included. This principle is followed in our crude model and we avoid the troubles of current algebra calculations obtaining good results for the $a_{1\pm}^{(-)}$ p wave scattering lengths.

Recently Höhler, Jakob and Strauss [89] systematically discussed various theoretical approaches to low energy parameters in comparison with dispersion relations. Their basic results agree with our conclusions.

CHAPTER IV

Quark diagrams and dual resonance models

In this chapter we extend the already introduced duality ideas in connection with the underlying quark rearrangements that take place in hadronic processes. The naive Feynman graphology, useful in evaluating decays of mesons and coupling constants, cannot be applied to more complex situations. On the other hand the dual relativistic quark models, using basically the Veneziano formula applied to quarks, while being very interesting as ideas are of limited practical applications.

So far the most successful formula in the field of Veneziano type models is the Lovelace formula [90] for $\pi\pi$ scattering. This formula having duality and crossing naturally built in, accounts, roughly, for the known physics of the $\pi\pi$ system in the whole range of energies. For detailed fits one requires unitarized versions of the model [91] but we shall not attempt to include such features in our discussion.

The attempts to apply Veneziano formula to other 4 particle amplitudes has not always met with complete success and certainly never achieved the general agreement with experiment and theory of the $\pi\pi$ formula. Trouble not only occurs, as we mentioned, with relativistic dual quark models but also with straight generalizations of the Veneziano formula. In πN scattering, for instance, those models lead to bad low energy parameters [45, 92].

Instead of imposing from the beginning a particular form for the amplitude we try, in this chapter, to extract from the Veneziano-Lovelace formula some basic properties. These properties, which are derived from the dual structure and not from the specific form of the Veneziano amplitude, are then classified and their consequences explored. We shall concentrate our attention only on scattering of pions on an arbitrary target T because pion mass extrapolations are small and allow direct comparison with soft limit results, but what we have to say could equally be extended to other pseudoscalar meson-target reactions.

We work in the framework of dual resonance models (*) and assume that planar duality holds, i. e., the scattering amplitude $A(s,t,u)$ has the quark underlying structure given by duality diagrams [43]. It can then be written as a sum of (s,t), (u,t) and (s,u) terms each one exhibiting poles in two channels, s and t, etc. According to the physical situation these quark model non exotic poles, are interpreted either as resonances or Regge poles. The terms $V(s,t)$ and $V(u,t)$ have the same quark structure and show no particular symmetry under $s \leftrightarrow t$, $u \leftrightarrow t$ interchanges respectively. The (s,u) diagram does not change when seem from s or u channels and the corresponding $U(s,u)$ term is then taken as even under $s \leftrightarrow u$ interchange:

$$U(s,u) = U(u,s). \tag{IV. 1}$$

Keeping in mind that a pure $U(s,u)$ term is t channel exotic $(I_t = 2)$ and the $s \leftrightarrow u$ crossing properties of the t channel isospin amplitudes we write the simplest and most general t channel isospin amplitudes for πT scattering in the form:

$$A_0^t = \beta [V(s,t) + V(u,t)] + \delta\, U(s,u) \tag{IV. 2 a}$$

$$A_1^t = \alpha [V(s,t) - V(u,t)] \tag{IV. 2 b}$$

$$A_2^t = \gamma\, U(s,u). \tag{IV. 2 c}$$

In principle $V(s,t)$ and $U(s,u)$ are not the same function and of the four Veneziano coefficients α, β, δ and γ two can be absorbed in the normalization of $V(s,t)$ and $U(s,u)$. Note that throughout this chapter we shall never assume a specific form for $V(s,t)$ and $U(s,u)$.

(*) For a review and definitions see Ref. [44].

We consider the reaction

$$\pi(q_1) + T(p_1) \to \pi(q_2) + T(p_2)$$

ann define the variable $\nu = q_1 \cdot (p_1 + p_2)/2\, m_T$, m_T being the mass of the target. We now assume that an expansion around $t=0$, $\nu=0$ is valid. At least in the πN case, as we have seen, such approximation is good. If in the region $\nu \simeq 0$ the s wave part of the amplitude is vanishingly small the s wave scattering lenght is approximately given by

$$a = m_\pi \left. \frac{\partial A(\nu, t=0)}{\partial \nu} \right|_{\nu \to 0} . \tag{IV. 3}$$

The best justification of (IV. 3) is the sucess of the current algebra calculations of scattering lengths (80, 83). Note that we are not strictly working in a soft pion limit. We assume that as far as the pion is concerned the soft limit results remain valid without drastic alternations when applied to physical situations because of the smallness of the pion mass.

Theoretically (IV. 3) is on more secure grounds for an isospin crossing odd amplitude because such an amplitude is constrained to vanish at $\nu = 0$ (*). This is in fact the amplitude we are most interested in in this chapter. We extend Equation (IV. 3) to each planar dual amplitude defining in this way a_{st}, a_{ut} and a_{su} contributions to the s wave scattering lengths. Using the $s \leftrightarrow u$ crossing properties of the t channel amplitudes we obtain from Equations (IV. 2) the following expressions for the scattering lengths:

$$a_0^t = \beta\,[a_{st} + a_{ut}] + \delta\, a_{su} = 0 \tag{IV. 4a}$$

$$a_1^t = \alpha\,[a_{st} - a_{ut}] = 2\,\alpha\, a_{st} \tag{IV. 4b}$$

$$a_2^t = \gamma\, a_{su} = 0\,. \tag{IV. 4c}$$

Note that if Equation (IV. 3) is true the addition of a Pomeranchuck like term to the amplitude would not affect the scattering

(*) For a discussion of σ terms and meson mass extrapolations see Ref. [93].

lengths because it is even under $s \leftrightarrow u$ crossing. For $\pi\pi$ scattering with physical pions the zero pion mass approximation of Equation (IV. 3) is not correct [80]. In particular, because of the additional symmetry, $a_{su} = a_{st}$, modifying Equations (IV. 4 a) and (IV. 4 c).

If we go now to the high energy limit in the forward direction

$$\operatorname{Im} A(s, t \simeq 0, u) \underset{s \to \infty}{\sim} \operatorname{Im} V(s, t \simeq 0) \qquad \text{(IV. 5)}$$

which implies that the resonance contributions from $\operatorname{Im} U(s, u)$ at $t \simeq 0$ do not add up at high energy to form Reggeons but rather compensate among themselves. Such compensations are achieved in chiral schemes [94, 95] as in the $\pi\pi$ Veneziano formula [90, 96, 97] by the inclusion of low lying particles (daughters). For example, in the $\pi\pi$ case the mass degenerate ρ and ε have equal and opposite contributions to $\operatorname{Im} U(s, u)$. We extend now these ideas to other processes.

As $\operatorname{Im} U(s, u)$ does not contribute at high energy in the forward direction one can write for $U(s, u)$ superconvergent relations in the form

$$\int \nu^k \operatorname{Im} U(s, u)\, d\nu = 0 \qquad (t \simeq 0) \qquad \text{(IV. 6)}$$

Equation (IV. 6) holds for all odd integers k. The most natural way of achieving this is by cancellations between high and low partial wave contributions in each local mass region. Then Equation (IV. 6) would be expected to hold also for even k. In our applications we restrict k to a value, $k = -2$, that provides convergence even for amplitudes which are not superconvergent and thus safely allows saturation with a few resonances. The test for superconvergence then becomes the local cancellation of the integral.

It is important to remark that the superconvergence of $U(s, u)$ is not derived here from exoticity in t channel but appears as a consequence of the dual planar structure of the amplitude. When an exotic t channel is present, $I_t = 2$, Equation (IV. 6) coincides with the superconvergent relations of Brout et al [97]. But, as shown below in the case of πN scattering, it is also valid when there are no exotic channels.

At this stage we compare our equations (IV. 1)-(IV. 6), which we think should be kept in a Veneziano formula for the πT scattering amplitude, with the Lovelace expression. Equations (IV. 1) and (IV. 2) are satisfied. Equations (IV. 3) and (IV. 4) are also satisfied up to terms in m_π^2 in the limit of linear expansion of the denominator Γ functions [96]. Equation (IV. 5) is obviously satisfied. Equation (IV. 6) is exact in the zero width resonance approximation. Note that the Veneziano formula for $K\pi$ scattering [98] also satisfies the equations that refer to $V(s,t)$ and $V(u,t)$ terms (there is no $U(s,u)$ term in πK scattering). In the case of the $\pi\eta$ system conditions (IV. 3) and (IV. 4) are not satisfied and the Lovelace formula is then not correct [99].

As the next step we discuss the consequences of imposing on the πT amplitude the constraints of the additivity quark model in the version proposed in previous work (Ref. [21] and chapter III of this thesis): quark model additivity is additivity of $V(s,t)$ π-quark duality diagrams generating the $V(s,t)$ πT diagram. We express the high energy additivity rule in the following way:

$$\begin{aligned} \mathrm{Im} < \pi T \,|\, A \,|\, \pi T > \underset{\substack{s \to \infty \\ t \simeq 0}}{\simeq} \sum_i \mathrm{Im} < \pi Q_i \,|\, A \,|\, \pi Q_i > = \\ = n \, \mathrm{Im}\, V_Q (s, t \simeq 0) \end{aligned} \tag{IV. 7}$$

where $\mathrm{Im}\, V_Q(s, t \simeq 0)$, a universal function of s, is the amplitude for the basic π-non strange quark Q interaction and n the number of interacting quarks in T (*). Via duality and Finite Energy Sum Rules (FESR) the high energy curve when extrapolated down to the low energy region must be, on average, equal to the low energy contributions. In this way the additivity rule (IV. 7) for the Imaginary part of the planar dual amplitude $V(s,t)$ can be extended to the whole range of energies.

We shall now apply these ideas to specific reactions and see how far they are satisfied in practice. We need to select an amplitude in which s channel resonances come only from $V(s,t)$, i.e., the A_1^t amplitude (Equation (IV. 2b)). As a «good»

(*) Note that the additivity rule (IV. 7) does not work for the real part of the amplitude because then $V(u,t)$ and $U(s,u)$ terms also contribute.

FESR, $(k = -2)$ we take the Adler-Weisberger relation [67, 68] interpreted as a FESR for A_1^t/ν^2 [100].

$$a_1^t = \left. \frac{A_1^t(\nu, 0)}{\nu} \right|_{\nu \to 0} = (\text{Born term}) + \frac{2}{\pi} \int_{\nu_{\text{th.}}}^{\infty} \frac{\operatorname{Im} A_1^t(\nu', 0)}{\nu'^2} d\nu'. \quad \text{(IV. 8)}$$

The sum rule is convergent which simplifies the comparison of different scattering processes because no cut-offs, which may be channel dependent, need be introduced.

To saturate the right hand side of (IV. 8) we generalize the procedure developed in Ref. [21] and in chapter III: in the resonance region we take only the first p wave resonances and treat the contributions above as high energy contributions, i. e., satisfying (IV. 7). We thus derive a set of SU (6) relations for coupling constants, by considering the p wave contributions to (IV. 8) of different πT reactions $(\pi Q, \pi\pi, \pi K, \pi N, \pi\Sigma, \pi\Xi)$ in the zero width approximation:

$$\left(\frac{g_{Q\pi}}{M_Q}\right)^2 = \left(\frac{g_{\rho\pi\pi}}{m_\rho}\right)^2 = \frac{8}{3}\left(\frac{g_{K^*K\pi}}{m_{K^*}}\right)^2 \quad \text{(IV. 9a)}$$

$$= \left(\frac{g_{N\pi}}{m_N}\right)^2 - \frac{16}{3}\left[\frac{g_{\Delta N\pi}}{m_N + m_\Delta}\right]^2 = \frac{1}{4}\left(\frac{g_{\Sigma\pi}}{m_\Sigma}\right)^2 \quad \text{(IV. 9b)}$$

$$+ \frac{1}{4}\left[\frac{g_{\pi\Sigma\Lambda}}{m_\Lambda + m_\Sigma}\right]^2 + \frac{2}{3}\left[\frac{g_{\Sigma^*\Sigma\pi}}{m_{\Sigma^*} + m_\Sigma}\right]^2 = \left(\frac{g_{\pi\Xi}}{m_\Xi}\right)^2 + \frac{8}{3}\left[\frac{g_{\Xi^*\Xi\pi}}{m_\Xi + m_{\Xi^*}}\right]^2$$

To obtain relations (IV. 9b) the kinematic factors appearing in the relativistic widths were approximated by putting $(m_\Delta + m_N)/ /m_N \simeq 2$ etc, as is usual in SU (6) calculations. Relations (IV. 9a) and (IV. 9b) are SU (6) as in the relativistic quark model [19]: the PPV (pseudoscalar, pseudoscalar, vector meson) coupling constant being proportional to m_V the BBP (baryon, barion, pseudoscalar meson) coupling being pseudovector in its nature with the s channel SU (3) mixing parameter $f^s \equiv F/(F + D) = 2/5$. We could write more SU (6) relations using target particles with higher spin (vector mesons, for instance) but then (IV. 7) should be interpreted in a spin average sense. Also if one substitutes a kaon for the pion more SU (6) relations are obtained. Note that we are not imposing the saturation of the Adler-Weisberger relation with p-wave resonances, but simply comparing their contributions in different processes.

As both the high energy and low energy contributions in the right hand side of (IV. 8) satisfy (IV. 7) obviously the left hand side also has to satisfy (IV. 7), i. e.:

$$a_1^t = n\, a \tag{IV. 10}$$

where a is a universal constant, the scattering length for πQ scattering. Equation (IV. 10) with Equations (IV. 4a) and (IV. 4c) reproduces Weinberg's universal scattering lengths for scattering of soft pions on any target [80].

The equations within each of the sets (IV. 9a) and (IV. 9b) are experimentally fairly well satisfied but the agreement is not so good when one equates meson to baryon coupling constants. The additivity relations provided by (IV. 7) are also not always well satisfied. They impose the condition of having pure F coupling in the t channel which is too strong. However our aim is not to check SU(6) but to stress that the vehicle for such an overall SU(6) consistent picture is the idea of duality. As emphasized several times by Rosner [55] duality is less restrictive than SU(6) or quark model additivity and it is probably more fundamental.

This is the point of view we take from now on when we consider the $U(s, u)$ integrals of Equation (IV. 6). We have another specific reason for doing so: the vanishing of these integrals cannot be achieved in the framework of SU(6) quark model $L = 0$ states as has been known for some time [94]. We are led back to the necessity of low lying particles to saturate (IV. 6). i. e. particles below the main trajectories initiated by the SU(6) $L = 0$ states. In first approximation we shall include in (IV. 6) all the observed p and s wave resonances in the first resonance region, in analogy with the ρ, ε case [94], and, because of the convergence argument referred to above, use $k = -2$ as in the Adler-Weisberger relation.

In general, from Equations (IV. 2), the s channel contributions of the $U(s, u)$ term can be isolated by the combination

$$U(s, u) \propto A_0^t - (\beta/\alpha)\, A_1^t \tag{IV. 11}$$

Equation (IV. 2) combined with the condition of no exotics in the s channel allows the following classification of the target particles according to their quark content:

Group 1 — Only one non strange quark in $T(K, \Xi, \cdots)$, $I_T = 1/2$

$$I_T = 1/2 : \delta = \gamma = 0 ; \ \beta/\alpha = 1 .$$

Group 2 — Two non strange quarks in $T(\pi, \Sigma, \wedge, \cdots)$, $I_T = 0, 1$

$$I_T = 0 : \gamma = 0 ; \ \alpha = 0$$

$$I_T = 1 : \delta/\gamma = -1/2 ; \ \beta/\alpha = 3/2 \qquad \text{(IV. 12)}$$

Group 3 — Three non strange quarks in $T(N, \Delta, \cdots)$, $I_T = 1/2, 3/2$

$$I_T = 1/2 : \gamma = 0 ; \ \text{No constraint on } \beta/\alpha$$

$$I_T = 3/2 : \delta/\gamma = -1 ; \ \beta/\alpha = 3 . \qquad \text{(IV. 13)}$$

Of course in Group 1 there is no $U(s, u)$ integral to satisfy, and from this point of view no low lying particles were required in these processes. It is perhaps not an unrelated coincidence that no low lying particles coupled to πK and $\pi \Xi$ have been unmistakably detected (101). The Ξ resonances, though not definitely classified, seem to fit quite well in the two main trajectories (102).

In Group 2 apart from $\pi\pi$ itself which leads to the results of Gilman and Harari (94) we can investigate Equation (IV. 6) for $\pi\Sigma$ scattering. The first p and s wave resonances are (*) $\wedge(1115)$, $\wedge(1405)$, $\Sigma(1189)$, $\Sigma(1385)$. Using Equations (IV. 11) and (IV. 12) we compute the left hand side of (IV. 6) for $k = -2$ in the zero pion mass limit to be

$$-(24 + 14) + (32 + 10) = -38 + 42 = (+4 \pm 8) \, GeV^{-2}$$

(*) The resonance parameters in $\pi\Sigma$ and πN scattering (mass, width and branching ratios) were taken from Refs. [103] and [101]. Except for the $\Delta(1236)$ resonance, where a Breit-Wigner formula with a $(q/q_\Delta)^3$ dependence in the width was used, the resonance contributions were evaluated in the zero width approximation. The widths were corrected for zero mass pions in the kinematic factors multiplying the coupling constants. The quoted errors simply include errors in the widths. For $f^s \equiv F/(F + D)$ we used the quark model value, $f^s = 2/5$.

which is compatible with zero on the right hand side. Our proposed mechanism of local mass cancellations seems then to work.

In Group 3 the testable case is πN scattering but here β/α is undetermined. Looking back to Equation (IV. 2), taking $t \simeq 0$ and the high energy limit one sees that β/α is related to $f^t \equiv F/(F+D)$ by [104]

$$\beta/\alpha = 4 f^t - 1 . \tag{IV. 15}$$

The first p and s wave resonances [105, 101] are now the $N(938)$, $N(1460)$, $N(1525)$, $\Delta(1236)$ and Equation (IV. 6) gives:

$$(1 - \beta/\alpha)(104 + 9.2 + 1.1) + (2 + \beta/\alpha)\, 60.8 = 0 \tag{IV. 16}$$

and, from (IV, 15),

$$f^t = 1.4 \pm 0.1 . \tag{IV. 17}$$

This value of f^t, larger than the SU(6) quark model value $f^t = 1$, is in reasonable agreement with the experimental determination and other theoretical predictions: $f^t \simeq 1.5$ (See Ref. [105] and further references there). If our arguments about the vanishing of $U(s, u)$ integral by cancellations in narrow mass strips are right, Equation (IV. 17) determines an high energy parameter from only a few low energy resonances. Because of the rapid convergence of (IV. 6) additional high energy contributions to (IV. 16) or to (IV. 14) would not change the results appreciably.

Returning to the s wave scattering lengths, deviations from the high energy quark model additivity as indicated by $f^t \neq 1$ are expected to cause violations in the universal scattering lengths. Taking the πN a_1^t scattering length as the standard quantity a the scattering lengths for $N\pi$, $\Sigma\pi$ and $\Xi\pi$ would be:

$$N\pi:\ a_1^t = a$$

$$\Sigma\pi:\ a_1^t = 2 f^t a \simeq 3 a \tag{IV. 19}$$

$$\Xi\pi:\ a_1^t = (2 f^t - 1)\, a \simeq 2 a \tag{IV. 19}$$

For $\Sigma\pi, \Xi\pi$ they are larger than predicted by universality. Neither (IV. 18) nor (IV. 19) can be unambiguously tested. It should be kept in mind that any result for scattering lengths relies on the validity of (IV. 3).

Other possible tests of Equation (IV. 6) are more speculative. However, we shall consider $\pi\rho$ and $\pi\Delta(1236)$ scattering using spin averaged amplitudes.

For $\pi\rho$, inserting the π, ω and A_1 poles, Equation (IV. 6) gives,

$$4\,\frac{g^2_{\rho\pi\pi}}{m^2_\rho} - 2\,g^2_{\omega\rho\pi} + \frac{(m^2_{A_1} - m^2_\rho)}{4\,m\,A_1^4}\,[2\,g_1^2 + g_0^2] = 0 \qquad \text{(IV. 20)}$$

where g_1 and g_0 are respectively the transverse and longitudinal couplings in the $A_1 \to \rho\pi$ decay, with

$$\Gamma_{A_1\rho\pi} = \frac{1}{12\,\pi}\,(2\,g_1^2 + g_0^2)\,q^5/m^2_{A_1}.$$

To keep consistency with our previous arguments and the local saturation of (IV. 6) with s and p waves only, the $A_1 \to \rho\pi$ decay should occur in a purely orbital s state. This corresponds to $|g_0/g_1| \approx 1$. (For information on the experimental situation and theoretical analysis of A_1 data, see Ref. [106]). To make an estimate of $\Gamma_{A_1\rho\pi}$ we allow ourselves some freedom in playing simultaneously with SU(6) and chiral symmetry. From $SU(6)_W$ we borrow the relation [40] $4\,g^2_{\rho\pi\pi}/m^2_\rho = g^2_{\omega\rho\pi}$ (*) and from chiral symmetry [94, 95] (or experiment), $m^2_{A_1} \approx 2\,m^2_\rho$. Neglecting terms in m^2_π, Equation (IV. 20) then gives:

$$\Gamma_{A_1\rho\pi} = \frac{1}{\sqrt{2}}\,\Gamma_{\rho\pi\pi} \qquad \text{(IV. 21 a)}$$

$$\simeq 90\,MeV. \qquad \text{(IV. 21 b)}$$

The width predicted in (IV. 21) is quite acceptable. Experimentally [101] $\Gamma_{A_1\rho\pi} \lesssim 95\,MeV$.

(*) This relation can be derived in the same way as relations (IV. 9 a) and (IV. 9 b) extracting from the Adler-Weisberger relation for $\pi\rho$ the p wave resonance contributions. It agrees approximately with the experimental width $\Gamma_{\omega\pi\gamma}$ with ρ dominance.

For $\pi\Delta$ scattering (IV.6), saturated with the same contributions as in the πN case (the $E(1525)$ is here negligible), allows a prediction for the $\Delta\Delta\pi$ coupling constant. We use data from Sutherland's work [107] and his definition of the $\Delta\Delta\pi$ coupling: $g_{\Delta^{++}\Delta^{+}\pi^{+}} \psi_{\mu}^{++} \gamma_5 \psi_{\mu}^{+} \varphi_{\pi}^{+}$. The result is

$$\frac{g^2_{\Delta^{++}\Delta^{+}\pi^{+}}}{4\pi} = 35^{+2}_{-5} \tag{IV. 22}$$

in good agreement with the SU(6) value [40,107], ≈ 32. In Ref. [107], from the Adler-Weisberger relation, a larger value is obtained but this, we think, is related to the general difficulty in saturating the Adler-Weisberger relation with a restricted number of resonances [108, 94, 100].

One could try to generalize these calculations to backward scattering. Instead of considering t channel Regge poles and s channel (or u channel) resonances one could also invert the procedure and consider t channel resonances and s channel (u channel) Regge poles. Regge poles in the t channel are expected to dominate at large s, small t, i. e., forward scattering. Regge poles in u channel should dominate at large t and $u \to 0$, i. e., backward or near backward scattering.

As we have seen, an important constraint which we impose on a dual resonance model is good behaviour both at threshold and in the high energy region. The threshold point, $t = 0$, $\nu = m_\pi$ is thus of particular interest. Going into the backward direction region, it is then natural to work exactly in the backward direction, i. e., $\cos\theta = -1$, and constrain the forward and backward amplitudes to be equal at threshold.

One should notice that the expansion of the amplitudes in the variable ν around the point $\nu \simeq 0$, $t \simeq 0$ does not necessarily have to be taken along the line $t = 0$. For instance in the original Weinberg's current algebra treatment [80] of low energy parameters one only requires $t = (q_1 - q_2)^2 = q_1^2 + q_2^2 - 2\, q_1 q_2$ being of order $0(m_\pi^2)$, i. e., of second order relative to the linear term in ν. This means that a backward expansion starting from the point $\nu = 0$, $t = 4\, m_\pi^2$ is also valid and should reproduce at threshold, $\nu = m_\pi$, $t = 0$ the same results of the forward expansion, i. e., Equation (IV. 3) and the current algebra scattering lengths. This is in fact another way of stating the condition of having $F_{\text{forward}} = F_{\text{backward}}$ at threshold.

Regarding Equation (IV. 6) the corresponding backward direction equation is:

$$\int t^k \operatorname{Im} V(s, t)\, dt = 0 \qquad (\cos\theta = -1). \tag{IV. 23}$$

The resonances to be included in (IV. 23) are the $\rho, \varepsilon, f_0, \rho'$ etc. Using in (IV. 23) local mass cancellations we come across the problem of the possible non existence of the ρ'. The ρ', predicted in the $\pi\pi$ Veneziano formula (90), is required to annihilate the f_0 contribution in the superconvergent sum rules. In the backward direction, this difficulty is in general more important because for the same value of k the integrals are less convergent than in the forward direction (ν increases faster than t as the masses of the resonances increase), and thus the role of higher mass resonances (f_0, ρ') is more relevant. In our calculations we limit ourselves again to the resonances in the first mass region, i. e., the ρ and the ε.

To isolate the t channel contributions of the $V(s, t)$ term we use, from Equations (IV. 2), the t channel isospin combinations,

$$V(s, t) \propto A_0^t + (\beta/\alpha)\, A_1^t \tag{IV. 24}$$

If we further simplify the problem by taking $m_\rho \simeq m_\varepsilon$ our saturation scheme gives, from (IV. 23) and (IV. 24),

$$\operatorname{Im} F_\varepsilon + (\beta/\alpha) \operatorname{Im} F_\rho = 0 \qquad (\cos\theta = -1) \tag{IV. 25}$$

It should be noticed that now, in contrast to what we had before, it is possible to write the $V(s, t)$ superconvergent relation (IV. 23) for πT processes belonging to any of the above referred Groups 1, II and III, provided the $V(s, t)$ term can be isolated via (IV. 24).

Equation (IV. 25) allows us to relate the coupling constants $g_{\rho TT}$ to $g_{\varepsilon TT}$. We work in the narrow resonance approximation using t channel elementary particle exchanges in the Feynman formalism. In $\pi\pi$ scattering there is a forward backward symmetry and Equations (IV. 11) and (IV. 24) are identical. Equation (IV. 25) in this case gives again the Gilman-Harari (94) result (*):

$$g_{\varepsilon\pi\pi}/g_{\rho\pi\pi} \simeq 1 \tag{IV. 26}$$

In πK scattering (IV. 25) gives:

$$g_{\varepsilon KK} = g_{\rho KK} \tag{IV. 27}$$

$$\simeq \frac{1}{2} g_{\varepsilon\pi\pi} \, . \tag{IV. 28}$$

Phase shift analysis of $K^+ p$ scattering provides an estimate of the quantity $g_{\varepsilon KK} g_{\varepsilon NN} \simeq 10$ (109). Comparing it to the corresponding πp product of coupling constants, $g_{\varepsilon\pi\pi} g_{\varepsilon NN}$ ($\simeq 50$ from various determinations), we obtain $g_{\varepsilon KK}/g_{\varepsilon\pi\pi} \simeq 1/5$ which is much smaller than (IV. 28) but has the same sign.

When the target T is a particle with spin the amplitudes in the forward and backward directions must have well defined spin properties. For targets with spin $\frac{1}{2}$ we use the t channel no-flip amplitudes. In the forward direction

$$F_f^{(\pm)} \equiv A'^{(\pm)} = A^{(\pm)} + \frac{\nu}{(1 - t/4\, m_T^2)} B^{(\pm)} \tag{IV. 29}$$

is related to the total cross sections and these are in fact the amplitudes used in previous forward direction calculations. In the backward direction we write the no-flip amplitudes (110):

$$F_b^{(+)} = A^{(+)}(s, -4\, q^2) + \frac{m_T\, \omega}{E} B^{(+)}(s, -4\, q^2) \tag{IV. 30}$$

$$F_b^{(-)} = \frac{m_\pi E}{m_T\, \omega} \left[A^{(-)}(s, -4\, q^2) + \frac{m_T\, \omega}{E} B^{(-)}(s, -4\, q^2) \right] \tag{IV. 31}$$

(*) The $\varepsilon\pi\pi$ and εKK coupling constants are defined using the Lagrangian:

$$L_\varepsilon = \frac{1}{2} g_{\varepsilon\pi\pi}\, m_\varepsilon\, \varepsilon\pi\pi + g_{\varepsilon KK}\, m_\varepsilon\, \varepsilon i K^+ \tau K .$$

where E, ω and q are center of mass variables, $E = \sqrt{q^2 + m_T^2}$, $\omega = \sqrt{q^2 + m_\pi^2}$. At $\cos\theta = -1$ also holds $t = -4q^2$ and $\nu = (s-u)/4 m_T = E\omega/m_T$. At threshold, as required, $F_b^{(\pm)} = F_f^{(\pm)}$. In πN scattering the ρ and ε contributions (*) to (IV. 25) are

$$\operatorname{Im} F_\rho^{(-)} = \frac{m_\pi}{4\pi} 2 \left[1 + K_N^V \frac{m_\rho^2}{4 m_N^2} \right] g_{\rho\pi\pi} g_{\rho NN} \pi \delta(t - m_\rho^2) \qquad \text{(IV. 32)}$$

and

$$\operatorname{Im} F_\varepsilon^{(+)} = -\frac{1}{4\pi} m_\varepsilon g_{\varepsilon\pi\pi} g_{\varepsilon NN} \pi \delta(t - m_\varepsilon^2) \qquad \text{(IV. 33)}$$

Similar contributions can be written for $\pi\Xi$ and $\pi\Sigma$ scattering. If one neglects the vector meson magnetic coupling contribution in (IV. 32) — in general this contribution is not negligible — and uses the values of β/α corresponding to pure F coupling in high energy t channel exchanges, Equation (IV. 25) would obviously give for the ε couplings the same SU(3) structure in πB as in πM scattering. In our calculations we shall keep the magnetic coupling contributions and use $f' = 1 \cdot 4$ (Equation (IV. 17)). To evaluate the $F_\rho^{(-)}$ resonances contributions we apply vector meson dominance, $g_{\rho BB} = I_B g_{\rho\pi\pi}$ and use for K_B^V the values given by the quark model, $F/(F+D) = \frac{3}{4}$ with $K_N^V = 3 \cdot 7$. From (IV. 25), (IV. 32) and (IV. 33) we obtain for the products of coupling constants $g_{\varepsilon\pi\pi} g_{\varepsilon BB}$ the values shown in Table IV. 1.

Having in mind the approximations involved the rough agreement with dispersion relations (111) in the εNN case is quite encouraging. If in (IV. 23) the f_0 is also included — it has to be if a ρ' effect does not exist — the values of the $g_{\varepsilon\pi\pi} g_{\varepsilon BB}$'s should be increased. Combining the result $g_{\varepsilon\pi\pi} g_{\varepsilon NN} \simeq 40$ with Equation (IV. 26) we obtain for the $g_{\varepsilon NN}$ coupling constant,

$$\frac{g_{\varepsilon NN}^2}{4\pi} \simeq 5.1\,. \qquad \text{(IV. 34)}$$

(*) The εNN coupling constant is defined from the Lagrangien $L_\varepsilon = g_{\varepsilon NN} \bar{N} N \varepsilon\,.$

TABLE IV.1

coupling constants:

$\varepsilon B B$	β/α	$g_{\varepsilon\pi\pi}\, g_{\varepsilon BB}$	
		Prediction	Dispersion Relations
$\varepsilon N N$	~ 5	~ 40	69 ± 4 Engels 63 Schlaile 50 Strauss
$\varepsilon \Xi \Xi$	1	$3 \cdot 5$	?
$\varepsilon \Sigma \Sigma$	3/2	10	?

Theoretical predictions on the εNN coupling constants (dispersion relations, dynamical equations with exchange potentials) give values for $g^2_{\varepsilon NN/4\pi}$ ranging from (112) 2.5 to 14.7.

In the forward direction we have seen that the $U(s,u)$ term corresponds either to a pure exotic t channel or to a t channel with degenerate trajectories cancelling among themselves. For instance the f_0 and ρ trajectoires are approximately degenerate and thus it is possible to write a forward «superconvergent» $U(s,u)$ relation for πN scattering. In the backward direction we expect the same situation to occur: the vanishing of the $V(s,t)$ integrals corresponds either to an exotic u channel or to a u channel with cancellations of trajectories. In $\pi\Xi$ and $\pi\Sigma$ $V(s,t)$ corresponds to $I_u = 3/2$ and 2, respectively. In πN there are no exotic channels and thus the «superconvergent» $V(s,t)$ relation to be valid requires cancellations of u channel trajectories. As the N_α and Δ_δ trajectories are not degenerate to achieve the required cancellations one needs to introduce two additional trajectories degenerate with the N_α and Δ_δ respectively. The need of at least four trajectories to satisfy duality in backward scattering is well known and is not surprising (115).

CHAPTER V

A few conclusions

We summarize now the main topics discussed in this thesis and draw some relevant conclusions.

In chapter I we enumerated a few problems in connection with a realistic interpretation of the quark model: the mass of the quark (free and bound quark), the binding potential, possible bootstrap schemes, dual models. These are the problems directly related to our work. Extremely interesting question like quark statistics, integrally charged quark models were completely left out. Concerning the quark statistics, at least as far as our work is concerned, we do not need to take up a position on this question. Concerning less economic quark models, using more than one SU(3) triplet, we think that the successes of duality and quark duality diagrams make them much less attractive than the one fractionally charged triplet quark model.

Cosmic rays experiments failed to confirm unambiguosly the «evidence for quarks». However a somewhat unexpected evidence for the realistic quark model appears to emerge from inclusive experiments. We cannot see a simple explanation for the presence of a «quark symmetry frame» based on pure (I, Y, B) and mathematical quark arguments.

In chapter II we developed a Bethe-Salpeter formalism applied to quarks. Not only mesons and baryons were described as bound state poles in the $\bar{Q}Q$ and QQQ channels respectively, but also their interactions were seen as built up from quark level interactions. SU(3) mass breaking factors are systematically introduced in various hadron coupling constants via the mass factors in the vector and pseudoscalar meson Bethe-Salpeter wave functions required by the constraint of vector meson dominance and the Goldberger-Treiman relation at quark level. These vertex functions are realizations of Llewellyn Smith Model I.

For baryons the Bethe-Salpeter formalism is complicated. However when non-relativistic approximations for the motion of the quarks are introduced we recover the usual results of the additivity quark model with corrections. In the evaluation of magnetic moments the predicted deviations from SU(6) are in fair agreement with experiment.

In chapter III we performed some quark level calculations (sum rules, dispersion relations) and showed that quarks, from a theoretical point of view, can be treated as strong interacting particles on the same footing as baryons and mesons. In particular the ideas of duality work for quark interactions in a way similar to that for hadrons. In fact, duality, with Harari-Rosner duality diagrams, can be seen as a new development of the quark model. We showed that the calculations of the old quark model, at low and high energy, are duality consistent. We used quark model low and high energy calculations to evaluate sum rules and dispersion relations finding good agreement with detailed calculations.

In chapter IV we explored the constraints derived from the quark rearrangements that occur in hadron reactions. Such constraints, we think, should be included in dual resonance models satisfying planar duality. One of the constraints takes the form of a superconvergent relation to be satisfied by local mass cancellattions. These «superconvergent» relations are more general than the usual ones because they do not require the presence of exotic channels. They are valid, for instance, in πN scattering. We made applications of these relations in forward and backward direction. In the backward directions we obtain a set of new predictions on the $\delta(750)$ coupling constants.

We think that as a whole our work brings support to a realistic interpretation of the quark model and shows that without any major difficulty quarks can be treated as dynamical objects in the (s, t, u) Mandelstam's plane.

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