THE INFLUENCE OF THE MIXING ON THE COULOMB EXCITATION PROBABILITIES OF BETA AND GAMMA BANDS (*)

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ABSTRACT — The influence of band-mixing on the excitation probabilities for multiple Coulomb excitation with ^{16}O ions for a gamma-band and a beta-band has been studied as a function of the energy of the ^{16}O projectile.

The case of a suggested beta-band in (d, d') work in ^{170}Er is used to show how multiple Coulomb exitation demonstrates the impossibility of that assignment in view of the discrepancy between the measured excitation probabilities and the calculated values.

1 — INTRODUCTION

Coulomb excitation is a time-dependent electromagnetic process of nuclear excitation produced by charged projectiles moving along orbits passing near the nucleus. To minimise the interaction due to nuclear forces there are restrictions on the maximum energy of the incident particle exciting the nucleus [1].

In the semiclassical approximation, the projectile orbit is treated classically, and the interaction with the nucleus quantum-mechanically.

The rapidly varying field, produced by the projectile, results in excitation of the nucleus.

The problem of finding the amplitudes of the excited nuclear states involves solving the time dependent Schrödinger equation [2]

$$i \hbar \frac{\partial |\psi>}{\partial t} = \{H_N + H_{int}(t)\} |\psi> \quad (1)$$

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where H_N is the Hamiltonian of the free nucleus and $H_{int}(t)$ is the Coulomb interaction energy between the nucleus and the projectile [3].

In radiative decay, magnetic transitions can compete with electric ones but not in the excitation process. In Coulomb excitation the magnetic field acting on the nucleus in a collision with the projectile is down by a factor v/c in relation to the electric field [3]. In addition, the magnetic interaction is proportional to the relative orbital angular momentum of the projectile-target system [3], and we shall assume scattering angles close to 180°. In fact we shall have in mind applications to beta and gamma bands in deformed nuclei in the region $150 < A < 190$ excited in experiments with ^{16}O ions back-scattered at an average angle of 160° [4], [5]. Therefore, only electric multipole matrix elements enter in the calculations.

Even if the increasing of multipole order decreases the Coulomb excitation probability much less than it decreases the rate of the inverse transition [3], one might think that most excitations would be of E1 type. In fact this is not so because the reduced matrix elements $< \alpha_f I_f \| \mathfrak{M}(E1) \| \alpha_i I_i >$ are very small and so very few E1 transitions have been observed. The most frequent excitation is of E2 type, and E3 excitations are commonly observed. Recently the effect of E4 moments on the Coulomb excitation of groud-state bands have been studied.

We have then

$$H_{int}(t) = 4\pi Z_P e \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \frac{1}{2\lambda+1} \frac{1}{r_P^{\lambda+1}} Y_{\lambda\mu}(\theta_P, \varphi_P) \mathfrak{M}^*(E\lambda, \mu) \tag{2}$$

with P referring to the projectile and

$$\mathfrak{M}(E\lambda, \mu) = \int \rho(\vec{r}) r^\lambda Y_{\lambda\mu}(\theta, \varphi) \, d\tau \tag{3}$$

the electric multipole moment of the nucleus.

The dependence of (1) on H_N is removed by passing to the interaction representation using the transformation

$$| \varphi > = e^{\frac{i}{\hbar} H_N t} | \varphi > . \tag{4}$$

Inserting in (1) one obtains

$$i\hbar\frac{\partial}{\partial t}|\varphi> = \tilde{H}(t)|\varphi> \tag{5}$$

with

$$\tilde{H}(t) = e^{\frac{i}{\hbar}H_N t} H_{int}(t)\, e^{-\frac{i}{\hbar}H_N t}. \tag{6}$$

The nuclear wave function $|\varphi>$ can be expanded in the complete set of eigenfunctions $|\alpha I M>$ of the undisturbed Hamiltonian H_N whose eigenvalues are $E_{\alpha I}$

$$|\varphi> = \sum_{\alpha I M} a_{\alpha I M}(t)\,|\alpha I M> \tag{7}$$

with α the remaining quantum numbers necessary to specific the nuclear state.

Substituting (7) in (5) we obtain a set of coupled equations equivalent to (5)

$$i\hbar\,\dot{a}_{\alpha I M}(t) = \sum_{\alpha' I' M'} e^{\frac{i}{\hbar}(E_{\alpha I} - E_{\alpha' I'})\,t} \tag{8}$$

$$<\alpha I M|H_{int}(t)|\alpha' I' M'>\, a_{\alpha' I' M'}(t)$$

where $a_{\alpha I M}(t)$ are the amplitudes of the nuclear state vector in the set of states of the undisturbed Hamiltonian. The problem is to find $a_{\alpha I M}(t = +\infty)$, integrating (8), with the initial conditions

$$a_{\alpha_i I_i M_i}(t = -\infty) = \delta_{\alpha_i \alpha_0}\,\delta_{I_i I_0}\,\delta_{M_i M_0} \tag{9}$$

with $\alpha_0 I_0 M_0$ the quantum numbers of the ground state.

The reduced matrix elements $<\alpha I\|\mathfrak{M}(E\lambda)\|\alpha' I'>$ are introduced in (8) using the Wigner-Eckart theorem.

Once the final amplitudes $a_{\alpha I M}(t = +\infty)$ on the states with spin I and magnetic quantum number M are known, one may easily obtain the probability for Coulomb excitation from the unpolarized ground-state $|\alpha_0 I_0 M_0>$ to the state $|\alpha I M>$ regardless of the orientation of the initial and final states

$$(10) \qquad P_{\alpha I} = \frac{1}{2 I_0 + 1} \sum_{M_0 M} | a_{\alpha I M} |^2 .$$

Therefore, with the use of heavy ions as projectiles, multiple Coulomb excitation of a nuclear state via various intermediate states becomes a very important tool for the study of transition matrix elements in the low energy region. In fact the excitation probabilities (10) through (8) and (2) depend on a set of matrix elements $<\alpha_f I_f || \mathfrak{M}(E\lambda) || \alpha_i I_i>$ between several states.

As we are interested in illustrating the contribution of multiple Coulomb excitation to clarify a consistent description of beta and gamma bands, we shall recall the main principles involved in the rotational model with first order band mixing, and refer to the calculation of reduced matrix elements using this model.

The deviations from the strong-coupling rotational model, in which the zero order Hamiltonian is separable into intrinsic and rotational parts, may be interpreted in terms of coupling between intrinsic motion and rotation. Bohr and Mottelson introduced the higher order terms as a power series in the total angular momentum to be treated by perturbation theory. For an axially symmetric nuclear shape, taking into account hermicity and invariance under time reversal and rotations about the body symmetry axis, one obtains

$$(11) \qquad H = H_{rot} + H_{int} + h_0 (I^2 - I_3^2) + h_{+1} I_- + $$
$$+ h_{-1} I_+ + h_{+2} I_-^2 + h_{-2} I_+^2$$

assuming higher powers of I_+ and I_- negligible to the order of interest. Here H_{rot} is the rotational Hamiltonian, H_{int} is the intrinsic Hamiltonian, $I_\pm$ are angular momentum operators in the body-fixed system which lower or raise K by one unit, while $h_{\pm i}$ are intrinsic operators that change the K of the intrinsic function by i units.

In even-even deformed nuclei we shall only consider the mixing of the ground-state $(K=0)$ band with the beta $(K=0)$ and gamma $(K=2)$ bands. We shall neglect couplings with higher-lying $K=1$ bands, i. e., we neglect the $h_{\pm 1}$ terms in (11). It is also assumed that the interaction is weak so that we can treat the coupling terms in (11) by first order perturbation theory.

The wave functions with first order admixtures for states in those bands, and the reduced matrix elements calculated with those wave functions were discussed elsewhere [1], [5], [6].

2 — MULTIPLE COULOMB EXCITATION CALCULATIONS IN THE SUDDEN APPROXIMATION

The presence of the exponential in (8) makes the solution of that set of coupled differential equations very difficult. Lütken and Winther [7] obtained solutions for (8), with model-dependent calculations, and avoided the difficulty of the exponential using the sudden approximation. All the nuclear states are considered as degenerate in this approximation.

The calculations, using the pure rotational model for axially symmetric deformed nuclei, without mixing, were particularly simple for backward scattering and give a qualitative picture of the excitation process. The backward scattering ensures that the magnetic quantum number is conserved during the collision. Lütken and Winther [7] tabulated, in table 4, the relative excitation probability for a level I for a given K band

$$|B^{\lambda}_{I,K}(q)|^2 = \frac{P_{I,K}}{\sum_I P_{I,K}} \tag{12}$$

as a function of a convenient parameter

$$q = 7.624 \frac{A^{\frac{1}{2}} Q_0}{\left(1 + \frac{A_1}{A_2}\right)^2 Z_1 Z_2^2} E^{\frac{3}{2}} \tag{13}$$

where the subscripts 1 and 2 refer, respectively, to projectile and target nucleus, Q_0 is the intrinsic quadrupole moment and E the projectile energy (MeV) in the laboratory system.

3 — A COMPUTER PROGRAM FOR MULTIPLE COULOMB EXCITATION CALCULATIONS

Winther and de Boer [8] wrote a computer program that integrates, on a model-independent way, the set of coupled differential equations (8). The program computes the excitation probabilities for a system consisting of a target nucleus with a finite number of levels and a projectile moving on a classical orbit. The program is independent of any nuclear model and provides a solution subject only to the limitations of the semiclassical approximation.

The imput of this program consists of the nuclear charges and masses, «spins» and energies of the nuclear levels, projectile energy and scattering angle, and the reduced matrix elements between all nuclear levels relevant to the calculations.

The extent of the dependence of the calculated excitation probability of a given level on a particular matrix element connecting any two particular levels may be studied varying this matrix element in the input data.

4 — RESULTS AND DISCUSSION

The values of the relative excitation probabilities for beta and gamma bands as a function of the ^{16}O incident energy are shown in Figs. 1 and 2. The nucleus ^{152}Sm was chosen as a suitable example because of its low-lying beta and gamma bands. However, the behaviour is very similar to the nuclei in the deformed region under consideration in this work.

To calculate the reduced matrix elements for the input data of Winther and the Boer program [8], we considered for the beta band the parameters

$$B(E2, 0_{gs} \rightarrow 2_{gs}) = (3.40 \pm 0.15)\, e^2 b^2$$

$$B(E2, 0_{gs} \rightarrow 2_{\beta}) = (2.28 \pm 0.17) \times 10^{-2}\, e^2 b^2$$

$$Z_0 = 0.08$$

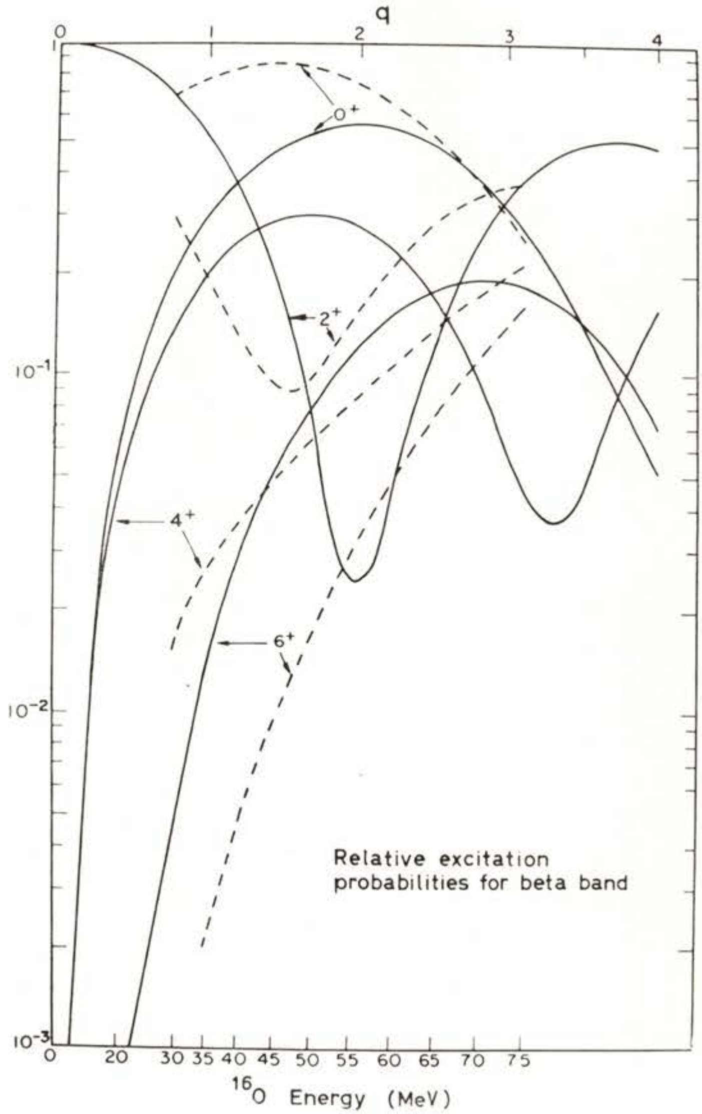

Fig. 1 — Calculated relative excitation probabilities for the 0^+, 2^+, 4^+ and 6^+ states of the beta band, in ^{152}Sm, for multiple Coulomb excitation with ^{16}O ions. The solid curves show the sudden approximation results of Lütken and Winther |7| and the dashed curves show the results of a calculation using the exact level energies and a value of $Z_0 = 0.08$ to acount for the mixing with the ground-state band.

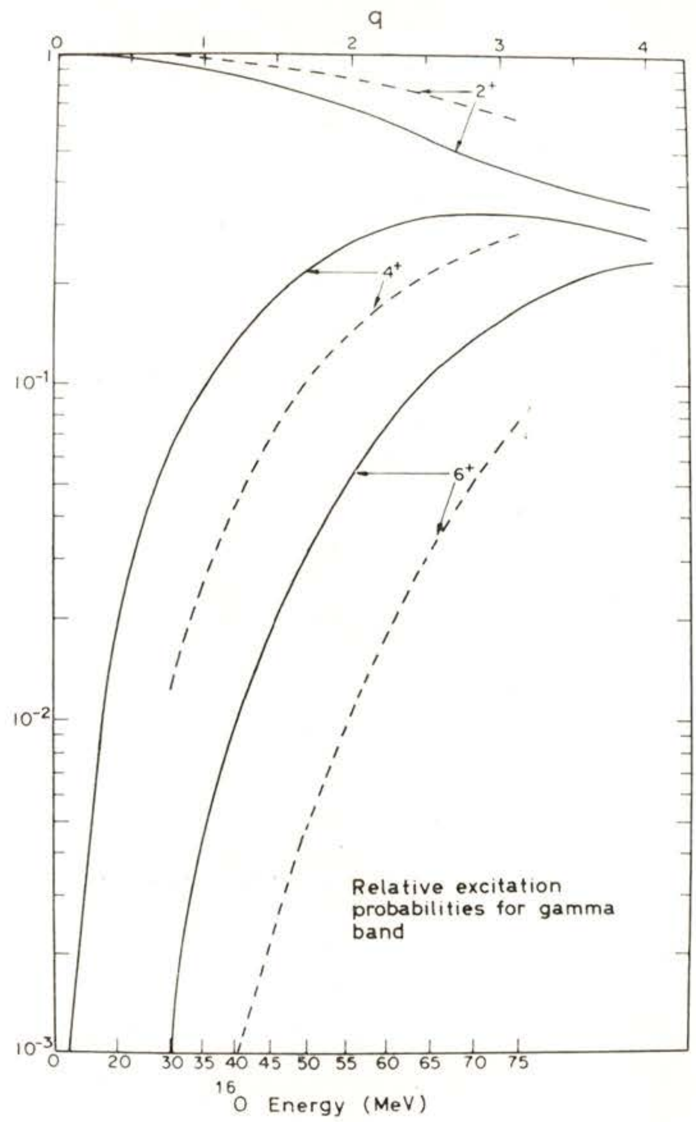

Fig. 2 — Calculated relative excitation probabilities for the 2^+, 4^+ and 6^+ members of the gamma band, in ^{152}Sm, for multiple Coulomb excitation with ^{16}O. The solid curves show the sudden approximation results of Lükten and Winther |7| and the dashed curves show the results of a calculation using the exact level energies and a value of $Z_2 = 0.08$ to account for the mixing with the ground-state band.

and for the gamma band

$$B(E2, 0_{gs} \rightarrow 2_{gs}) = (3.40 \pm 0.15)\, e^2 b^2$$

$$B(E2, 0_{gs} \rightarrow 2_{\gamma}) = (8.13 \pm 0.57) \times 10^{-2}\, e^2 b^2$$

$$Z_2 = 0.08\,.$$

The meaning of the mixing parameters Z_0 and Z_2 and the procedure involved in calculating the matrix elements, relevant to these calculations, can be seen in Sec. III and IV of ref. [5].

The 3^+ and 5^+ levels of the gamma band are omitted in Fig. 2 because states of unnatural parity are not excited to any appreciable extent in backward scattering.

For comparison the sudden approximation values, for pure bands, are also shown in Figs. 1 and 2.

It should be noted that in the energy range of interest here, i. e., 40-60 MeV, the excitation of the 2^+ state and higher states in the beta band is predicted to be small compared with that of the 0^+ state.

Tjøm and Elbek [9] studied the nucleus ^{170}Er in inelastic deuteron scattering, and suggested that a 0^+ state at 880 keV and a 4^+ state at 1122 keV together with a 2^+ state at 959 keV could be members of a $K = 0$ beta band. That same nucleus was studied in Coulomb excitation with ^{16}O by J. M. Domingos et. al. [4] and transitions were found that are consistent with a 0^+ state at 889 keV and a 4^+ state at 1124 keV. However they found no evidence for a 2^+ state as proposed by Tjøm and Elbek. If we think in terms of the relative excitation probabilities, as shown in Fig. 1, we might think that the absence of that 2^+ state, in a Coulomb excitation experiment at a ^{16}O effective energy of 53.5 MeV, would not contradict, in principle, their assignment for those states as members of a beta band. A step further could be to assume a ground-state band as shown in ref. [4] and a beta band with 0^+, 2^+ and 4^+ states at 0.889 MeV, 0.959 MeV and 1.124 MeV and an additional 6^+ state at a predictable energy of 1.36 MeV, to perform theoretical calculations with the Winther and de Boer [8] computer program. The comparison of the experimental values with the excitation probabilities calculated for such a beta band, as shown in Table 1 for no mixing ($Z_0=0$) and for a reasonable mixing $Z_0=0.04$, clearly excludes that assignment.

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TABLE 1. Comparison with experimental results of calculated excitation probabilities P_I for an effective energy of 53.5 MeV ^{16}O ions for a hypothetical beta band in ^{170}Er. Values of the mixing parameter $Z_0=0$ (no mixing) and $Z_0=0.04$ were assumed.

	Calculated values		Experimental values
	$Z_0=0$	$Z_0=0.04$	
P_0+	0.00205	0.00211	0.00030±0.00003
P_2+	0.00005	0.00023	—
P_4+	0.00074	0.00030	0.00049±0.00004
P_6+	0.00029	0.00007	—

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