THE FIELD OF THE DIVERGENCE OF ENTHALPY OF THE ATMOSPHERE IN THE SOUTHERN HEMISPHERE (*)

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SUMMARY — A study of the mean atmospheric divergence of enthalpy fields on a planetary scale for the Southern Hemisphere during the IGY covering the calendar year 1958 is presented. The fields of divergence of enthalpy integrated along the vertical for yearly and seasonal conditions for the entire hemisphere are analysed and discussed. In order to obtain a view of the distribution of the enthalpy along the vertical, zonally averaged values of the enthalpy at various levels up to 50 mb were also computed. The mean zonal results of the vertically integrated values are shown and the zonal estimates were compared whenever possible with previous results and with climatological data. In particular the divergence of the various modes and their partial contribution for the mean total divergence are fully discussed. The structure of these fields is studied and the corresponding implications for the energetics of the atmosphere mainly for the energy budget are discussed. The influence of oceans in establishing the energy balance is mentioned. The consequences for the general circulation are analysed regarding the divergence map as a distribution of the heat sources and sinks on a global scale, as a basis for the analysis of the physical mechanisms which lead to the conversion of the various forms of energy.

1 — INTRODUCTION

The present study may be regarded as an extension and an application of the theory developed in a previous paper by the writer (Peixoto, 1974).

Diabatic heating in the atmosphere occurs as a result of the convergence of radiative flux, of phase changes of water substance

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and of the heat exchange between the earth and the atmosphere. Solar radiation is the ultimate source of radiative energy, whereas thermal or terrestrial radiation constitutes a sink. The release of latent heat is the predominant diabatic heating effect in the troposphere above the atmospheric boundary layer. The turbulent heat exchange near the earth's surface in the boundary layer is generally from the earth to the atmosphere because the ocean surface is usually warmer than the air above and in addition in the tropical zones and in the summer hemisphere at middle latitude regions the land surfaces are also relatively warmer.

However, south of 60° S, where oceans of near-freezing temperature, and floating ice of melting temperature, cover most of the area, heat must be given off from the atmosphere to the ocean. Much of the heat loss of atmosphere south of the Antarctic Circle may be represented by heat transfer to the melting snow and ice surfaces.

As discussed in the above mentioned paper (Peixoto, 1974) the divergence and convergence of enthalpy in the atmosphere plays a very decisive role in the energetics of the general circulation and the energy balances of the earth as a whole, and of the atmosphere and oceans systems, taken separately.

We must recognize that in studying the energy balance of a given region, the divergence of enthalpy is much more important than the enthalpy flux by itself. In fact, on a long-term basis the energy storage rate of change in the earth-atmosphere system can be assumed not to change substantially. Therefore, in order to keep the long range observed steady state of the atmosphere in regions where there is a net diabatic effect, mainly due to radiation (incoming minus outgoing radiation) release of latent heat, contact heat, etc., an outflow of energy carried by both the atmosphere and the oceans must be observed. On the other hand, whenever a deficit of the heat balance is observed an inflow of energy must take place. The substantial part of the mechanism which provides the necessary compensation is provided by the divergence field of enthalpy of the atmosphere.

On the whole the heating or the cooling due to the convergence or the divergence of enthalpy acts in a sense opposite to the diabatic effects in a given region (Peixoto, 1974).

Regions where the divergence of enthalpy is positive, $(\overline{\mathrm{div}\,\vec{S}} > 0)$, constitute, in the mean, sources of enthalpy. We conclude then, that in these regions there must be a net excess of energy when all the diabatic effects, such as radiation, release of latent heat, heat of contact, etc., are taken into consideration. This excess is, thus, exported under the form of enthalpy.

Inversely, regions where convergence of enthalpy, $(\overline{\mathrm{div}\,\vec{\mathrm{S}}<0})$, are predominant, constitute sinks of enthalpy, which determine an inflow of energy into such regions to compensate the observed deficit of energy.

There have been numerous studies concerned with the general circulation and the heat and energy balances of the atmosphere, mainly based on Northern Hemisphere data. Not until the International Geophysical Year (IGY), have sufficient data become available in the Southern Hemisphere because of the poor aerological network, due to vast ocean areas, to undertake this type of study.

Thus, the purpose of the present study is to use the IGY data within the framework of the observational approach to investigate seasonal changes in the divergence field of enthalpy in the atmosphere and its application to the energy budget of the earth, ocean and atmosphere, in the Southern Hemisphere. In previous papers seldom there have been attempts to use observations in the global circulation studies over the Southern Hemisphere and, therefore, no basis for comparing results obtained extensively for the Northern Hemisphere.

2 — FORMULATION OF THE PROBLEM

Since the formulation principles of the problem have been presented in a previous paper (Peixoto, 1974) only the basic equations will be given here.

For a unit column of the atmosphere, which extends from the surface $(p = p_0)$ to the top of the atmosphere $(p = 0)$ the balance equation of the total time mean enthalpy, $\overline{\mathrm{H}}$, is given by

$$\overline{\frac{\partial \mathrm{H}}{\partial t}} + \mathrm{div}\,\overline{\vec{\mathrm{S}}} = \overline{\{\dot{\mathrm{Q}}\}} \tag{1}$$

where the bar operator indicates a time average for the time interval τ

$$\overline{(\quad)} = \frac{1}{\tau}\int^{\tau} (\quad)\,dt\,.$$

Let us discuss the meaning of the symbols. The total enthalpy H is given by

$$\mathrm{H} = \frac{\mathrm{C}_p}{g} \int_0^{p_0} \mathrm{T}\, dp \tag{2}$$

where C_p denotes the specific heat at constant pressure, g the acceleration of gravity, T the temperature and p the pressure.

The transport field of enthalpy $\vec{\mathrm{S}}(\lambda, \Phi)$ is a two-dimensional vectorial field represented by

$$\vec{\overline{\mathrm{S}}} = \overline{\mathrm{S}}\ \vec{i} + \overline{\mathrm{S}}_\Phi \vec{j} \tag{3}$$

where $\vec{i}$ and $\vec{j}$ are the unit vectors of the local β-plane and $\overline{\mathrm{S}}_\lambda$ and $\overline{\mathrm{S}}_\Phi$ the zonal and the meridional components, respectively defined by

$$\begin{aligned} \overline{\mathrm{S}}_\lambda &= \frac{\mathrm{C}_p}{g} \int_0^{p_0} \overline{u\mathrm{T}}\, dp \\ \overline{\mathrm{S}}_\Phi &= \frac{\mathrm{C}_p}{g} \int_0^{p_0} \overline{v\mathrm{T}}\, dp \end{aligned} \tag{4}$$

where u and v are the zonal and the meridional components of the wind. $\{\dot{\mathrm{Q}}\}$ represents the total diabatic effects for all the column; $\{\dot{\mathrm{Q}}\}$ is the resultant of the net radiation (solar minus terrestrial), $\{\dot{\mathrm{Q}}_\mathrm{R}\}$, of the release of latent heat $\{\dot{\mathrm{Q}}_\mathrm{L}\}$, of the conduction of sensible heat from the ground $\{\dot{\mathrm{Q}}_\mathrm{C}\}$ and of friction $\{\dot{\mathrm{Q}}_\mathrm{F}\}$:

$$\{\dot{\mathrm{Q}}\} = \{\dot{\mathrm{Q}}_\mathrm{R}\} + \{\dot{\mathrm{Q}}_\mathrm{L}\} + \{\dot{\mathrm{Q}}_\mathrm{C}\} + \{\dot{\mathrm{Q}}_\mathrm{F}\}\,. \tag{5}$$

Equation (1) can be written explicitly in a spherical coordinates referencial (λ, Φ, p, t), where Φ and λ are the latitude and the longitude, as follows:

$$\frac{\overline{\partial \mathrm{H}}}{\partial t} + \frac{1}{a \cos\Phi} \left\{ \frac{\partial \overline{\mathrm{S}_\lambda}}{\partial \lambda} + \frac{\partial}{\partial \Phi} (\overline{\mathrm{S}}_\Phi \cos\Phi) \right\} = \{\overline{\dot{\mathrm{Q}}}\} \tag{6}$$

a is the mean radius of the earth.

The application of the Ostrogradsky-Gauss theorem to equation (1) leads to the equivalent equation

$$\frac{\overline{\partial H}}{\partial t} + \frac{1}{A} \oint (\vec{\overline{S}} \cdot \vec{n})\, d\, c = \{\overline{\dot{Q}}\} \tag{7}$$

when an area A is bounded by a contour c, with a normal vector $\vec{n}$. For an area bounded by the latitudes Φ and $\Phi + \Delta \Phi$ this equation transforms into

$$\frac{\overline{\partial H}}{\partial t} + \frac{1}{a \cos \Phi} \frac{\partial}{\partial \Phi} \oint \overline{S}_\Phi (\lambda, \Phi) \cos \Phi \, d\lambda = \{\overline{\dot{Q}}\} \tag{8}$$

where only the meridional convergence of enthalpy appears, since

$$\oint \frac{\partial S_\lambda}{\partial \lambda} d\lambda = 0 \, .$$

Expressions (4) can be expanded in the space-time domain using the bar and the «bracket-operator» [ ], given by:

$$[\quad] = \frac{1}{2\pi} \oint (\quad) d\lambda$$

as follows:

$$\begin{aligned} [\overline{S}_\lambda] &= \frac{C_p}{g} \left\{ \int_0^{p_0} [\overline{u}][\overline{T}]\, dp + \int_0^{p_0} [\overline{u' T'}]\, dp + \int_0^{p_0} [\overline{u}^* \, \overline{T}^*]\, dp \right\} \\ [\overline{S}_\Phi] &= \frac{C_p}{g} \left\{ \int_0^{p_0} [\overline{v}][\overline{T}]\, dp + \int_0^{p_0} [\overline{v' T'}]\, dp + \int_0^{p_0} [\overline{v}^* \, \overline{T}^*]\, dp \right\} . \end{aligned} \tag{9}$$

Symbolically we have then:

$$\begin{aligned} [\overline{S}_\lambda] &= \overline{S}_{\lambda M} + \overline{S}'_\lambda + \overline{S}^*_\lambda \\ [\overline{S}_\Phi] &= \overline{S}_{\Phi M} + \overline{S}'_\Phi + \overline{S}^*_\Phi \, . \end{aligned} \tag{9a}$$

These expressions show that the total mean fluxes of enthalpy are carried by the mean zonally averaged motion (mean circulations) $\vec{\overline{S}}_M$, by the transient eddies $\vec{\overline{S}}'$, and by the standing eddies $\vec{\overline{S}}^*$.

In a global scale the mean divergence of enthalpy is given by:

$$\operatorname{div}[\overline{\vec{S}}] = \operatorname{div}\overline{\vec{S}}_M + \operatorname{div}\overline{\vec{S}'} + \operatorname{div}\overline{\vec{S}^*}. \tag{9b}$$

The first term of R. H. S. of equations (9) or (9a) is the advection of mean temperature field due to the mean zonally averaged wind field. The second term associated with the zonal average along a latitude circle of the time correlation of winds and temperatures at individual points measures the transport associated with the transient horizontal eddies (baroclinic perturbations, cyclones, etc.).

The last term measures the zonally averaged flux associated with the standing eddies (semi-permanent lows, anticyclones, etc.).

For a long period of time the local rate of change of storage of enthalpy in the atmosphere is negligible and in the balance equations we may set it at zero:

$$\frac{\overline{\partial H}}{\partial t} = 0. \tag{10}$$

From equation (1) under steady state conditions the divergence of enthalpy equals the diabatic effects in such a way that there is compensation of cooling $\{\overline{\dot{Q}}\} < 0$ by convergence of enthalpy, $\overline{\operatorname{div}\vec{S}} < 0$, and excess of warming $\{\overline{\dot{Q}}\} > 0$ by export of enthalpy, $\overline{\operatorname{div}\vec{S}} > 0$. The sources of enthalpy are associated in the mean with the warming due to diabatic effects, and the sinks with the corresponding cooling of the atmosphere.

Let us focus our attention on expressions (9) and (9a). For the global budget of energy of the system oceans-atmosphere the meridional transport is more relevant than the zonal transport. So we will analyse with some more detail the various components of $[\bar{S}_\Phi]$. The flux of enthalpy by the mean meridional circulation, $\bar{S}_{\Phi M}$, is given by:

$$\bar{S}_{\Phi M} = \frac{C_p}{g} \int_0^{p_0} [\bar{\nu}][\bar{T}]\, dp. \tag{11}$$

As it is well known it is very difficult to estimate mainly due to the uncertainty and lack of precision involved in the evaluation of the mean meridional circulation $[\bar{\nu}]$. If the winds were geostrophic the mean zonal value of $[\bar{\nu}_{gs}]$ would vanish identically.

However, with actual winds $\overline{S}_{\Phi M}$ has not to be zero. The corresponding values are associated with the transport of enthalpy by the mean meridional circulations. This transport may be important in the tropical regions where the Hadley cell predominates, whereas the mean meridional eddy transports are more important in middle and high latitudes, the transient eddies being more predominant. This point will be taken again, later.

3 — DATA AND PROCEDURES

The data used in this study are the values of $\overline{S}'_\lambda$ and $\overline{S}'_\Phi$ at grid points with 5° latitudinal and longitudinal increments over the Southern Hemisphere between 0° and 80° S. These values were read from maps with the analyses of $\overline{S}'_\lambda$ and $\overline{S}'_\Phi$ fields (Peixoto, 1973) obtained from directly observed values of u, v and T at various isobaric levels during the IGY for 125 upper-air stations which form the basic aerological network. The maps were analysed for the winter season (April-Sept), for the summer season (Oct-March) and for the whole year.

The bidimensional continuum (λ, Φ) is substituted by a discretum (λ_j, Φ_j) in which the dimensions of the fundamental grid are $\delta\,\Phi_j = \delta\,\lambda_j = \dfrac{\pi}{36}$ and $\lambda_j = j\,\dfrac{\pi}{36}$, $\Phi_j = j\,\dfrac{\pi}{36}$ where j is an integer number varying from 0 to 72 for the longitude λ and from 0 to 16 for the latitude Φ.

In a generic point (λ_k, Φ_k) of the discretum the expression of mean bidimensional (lateral) divergence of enthalpy is:

$$\operatorname{div}\overline{\vec{S}}' = \frac{1}{a\cos\Phi_{k+\frac{1}{2}}}\left[\frac{\partial\,\overline{S}'_{\lambda_k}}{\partial\,\lambda_k} + \frac{\partial\,(\overline{S}'_{\Phi_k}\cos\Phi_k)}{\partial\,\Phi_k}\right]. \tag{12}$$

Using centered differences we arrive at the expression in finite differences:

$$\operatorname{div}\overline{\vec{S}}' = \frac{1}{a\cos\Phi_{k+\frac{1}{2}}}\,\frac{36}{\pi}\,[\delta\,\overline{S}'_{\lambda_k} + \delta\,\overline{S}'_{\Phi_k}\cos\Phi_k]\,. \tag{13}$$

The centered differences $\delta \overline{S}'_{\lambda_k}$ and $\delta \overline{S}'_{\Phi_k} \cos \Phi_k$ using current notation are given by

$$\begin{aligned} \delta \overline{S}'_{\lambda_k} &= \overline{S}'_{\lambda_{k+\frac{1}{2}}} - \overline{S}'_{\lambda_{k-\frac{1}{2}}} \\ \delta \overline{S}'_{\Phi_k} \cos \Phi_k &= \overline{S}'_{\Phi_{k+\frac{1}{2}}} \cos \Phi_{k+\frac{1}{2}} - \overline{S}'_{\Phi_{k-\frac{1}{2}}} \cos \Phi_{k-\frac{1}{2}} \, . \end{aligned} \tag{14}$$

Numerical computation of divergence in a region bounded by a contour c becomes much simplified when the contour c is formed by segments of meridians and paralels, because disposing of all the grid points values of the discretum the boundary values are immediately available.

If, for instance, the contour c is formed by two segments of paralels $\Phi_1 = \text{const.}$ and $\Phi_2 = \text{const.}$ of amplitude $n \delta \lambda = \lambda_2 - \lambda_1$ and by two segments of meridians $\lambda_1 = \text{const.}$ and $\lambda_2 = \text{const.}$ of amplitude $m \delta \Phi = \Phi_2 - \Phi_1$ where m and n are integer numbers, the arithmetic expression of the total enthalpy flux across c is:

$$\begin{aligned} \oint (\vec{\overline{S}}' \cdot \vec{n}) \, dc = a \frac{\delta \Phi}{2} \Bigg\{ & (\overline{S}'_{\lambda_{2,0}} - \overline{S}'_{\lambda_{1,0}}) + 2 \sum_{\alpha=1}^{m-1} [\overline{S}'_{\lambda_{2,\alpha}} - \overline{S}'_{\lambda_{1,\alpha}}] + \\ & + (\overline{S}'_{\lambda_{2,m}} - \overline{S}'_{\lambda_{1,m}}) \Bigg\} + a \frac{\delta \lambda}{2} \Bigg\{ (\overline{S}_{0,\Phi_2} \cos \Phi_2 - \overline{S}'_{0,\Phi_1} \cos \Phi_1) + \\ & + 2 \sum_{\beta=1}^{n-1} [\overline{S}'_{\beta,\Phi_2} \cos \Phi_2 - \overline{S}'_{\beta,\Phi_1}] + (\overline{S}'_{n,\Phi_2} \cos \Phi_2 - \overline{S}'_{n,\Phi_1}) \Bigg\} . \end{aligned} \tag{15}$$

If the contour c bounds a nonsimply connected spherical zone, limited by two paralels $\Phi_1 = \text{const.}$ and $\Phi_2 = \text{const.}$, the first term of the second member of the preceding expression is zero and, taking into consideration the dimensions of the grid (for the entire paralel $n = 72$), the final expression for computational purposes becomes:

$$\oint (\vec{\overline{S}}' \cdot \vec{n}) \, dc = a \, \delta \lambda \sum_{\beta=1}^{72} [\overline{S}'_{\beta,\Phi_2} \cos \Phi_2 - \overline{S}'_{\beta,\Phi_1} \cos \Phi_1] \, . \tag{16}$$

The area A bounded by the contour c is now given by $A = 72\, a^2 \cos \Phi_{1+\frac{1}{2}} \,\partial \lambda\, \partial \Phi$ and the corresponding mean value of divergence is:

$$\operatorname{div} \overline{\vec{S}'} = \frac{1}{A} \oint \overline{\vec{S}'} \cdot \vec{n}\, dc =$$

$$= \frac{1}{a \cos \Phi_{1+\frac{1}{2}} \,\partial \Phi} \sum_{\beta=1}^{72} [\overline{S}'_{\beta, \Phi_2} \cos \Phi_2 - \overline{S}'_{\beta, \Phi_1} \cos \Phi_1] . \tag{17}$$

The mathematical treatment for the computation of the field of divergence of enthalpy and the processing of the basic data was done with the help and cooperation of the M. I. T. Planetary Circulations Project and with the Numerical Weather Prediction Unit of the National Meteorological Service of Portugal.

With the computed values of the mean horizontal divergence of enthalpy, $\overline{\operatorname{div} \vec{S}'}$, in approximately 648 points, for the year, summer and winter semesters, hemispheric charts were prepared with the mean values of divergence referred to the centers of each element of the grid of the discretum. An analysis of the divergence of enthalpy field was then performed for the mentioned periods, using the scalar analysis methodology by the drawing of isopleths.

4 — ANALYSIS AND INTERPRETATION OF RESULTS

4. 1. *Analysis of the hemispheric fields of divergence*

The maps with the analysis of the field of mean divergence of the vertically integrated transient eddy flux of enthalpy relative to the year and to the winter and summer semesters, for the Southern Hemisphere are represented in figures 1, 2 and 3.

The isopleths are in units of 10^{-3} cal/(cm$^2 \cdot$ min) that is, approximately 1,44 langley/day (ly/day). The isopleths of positive divergence are drawn as continuous lines while those for convergence are dashed lines.

The exam of the spacial distribution of the divergence field shows that in any of three maps the regions of convergence and those of divergence are not formed by isolated centers. The analysis present an almost continuous zonal distribution with some sort of polar

Fig. 1 — Mean annual distribution of the horizontal divergence of the vertically integrated transient eddy flux of enthalpy in 10^{-3} calories per square centimeter per minute.

symmetry although with some variable longitudinal intensities. It is apparent that, in middle latitudes, most part of oceans, are zones of divergence (lost of enthalpy), distributed in centers of greater intensity mainly in the zone between 20° and 45° S. On the other hand zones of convergence predominate south of the latitude circle of 50° S. There is also slight convergence in some equatorial regions.

The analysis of the map relative to the year shows that in Southern Hemisphere the areas of regions of convergence and the

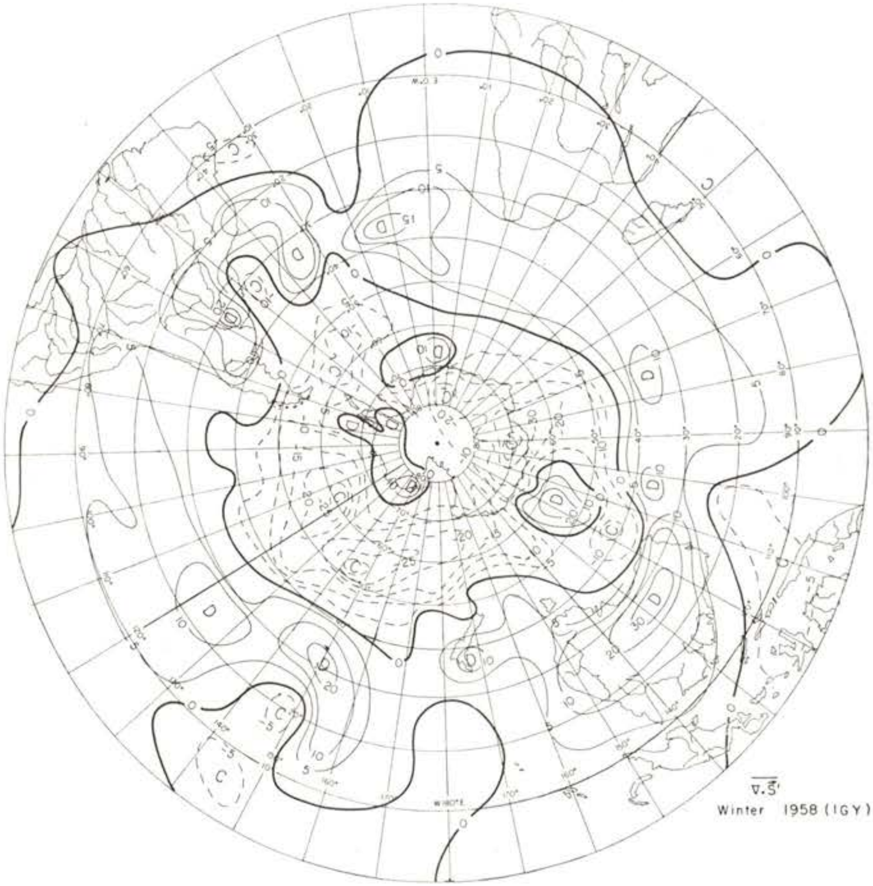

Fig. 2 — Mean winter distribution of the horizontal divergence of the vertically integrated transient eddy flux of enthalpy in 10^{-3} calories per square centimeter per minute.

areas of divergence of enthalpy associated to the eddies almost balance themselves. This is due to the fact that the net mean annual eddy flux of enthalpy across the equator is almost zero. This aspect will be discussed later with more detail.

Consequently for the interval of a year, regions where there is an excess of energy must, on the average, balance those where a deficit is observed.

Fig. 3 – Mean summer distribution of the horizontal divergence of the vertically integrated transient eddy flux of enthalpy in 10^{-3} calories per square centimeter per minute.

Thus, for a sufficiently long interval of time the storage power of atmosphere for the enthalpy cannot suffer an appreciable variation. This leads to the conclusion that the annual variation of the enthalpy stored in the atmosphere is very small.

The centers of convergence are located in the equatorial regions and in middle and high latitudes, south of the latitude 50° S, with the exception of small centers of divergence in the polar regions.

Centers of divergence are located in the east position of Brasil and Argentina extending over most of the South Atlantic Ocean, over South Africa and over Australia. However the most important centers of divergence are located over the oceans. The more intense are observed over the Atlantic near the South America Coast over a large belt covering all the central Pacific and another belt over the Indian Ocean. It is interesting to point out that the regions where divergence is most intense are located in the latitudes where the circulation of the atmosphere is associated with the large semi-permanent subtropical anticyclones.

The most predominant centers of divergence, over the Continents coincide with arid or desertic regions such as happens with Australia. Therefore, the regions of large subtropical anticyclones which are related to the location of desertic regions, are sources of enthalpy for the atmosphere. This is to be expected, since the subtropical oceanic zones are regions of formation of tropical maritime air masses (Tm), caracterized by high temperature and humidity. Furthermore it is interesting to see that the dynamics of climate plays a predominant role in the modelation of the crust and in the physiography of the globe.

The existence of divergence of enthalpy over the ocean agrees with the fact that oceans constitute sources of heat for the atmosphere due to its large capacity of storing enthalpy and to the small fluctuations of the mean temperature of water. Also, in the region of divergence, the evaporation is stronger, which is corroborated by the higher values of the salinity in that regions. It is also interesting to stress that in the Southern Hemisphere the Continental zones, in spite of some isolated centers of convergence, show, on the average, a net divergence of enthalpy during all the year.

The maps with the seasonal analyses of the divergence of enthalpy (figs. 2 and 3) show that the global configurations of the corresponding seasonal fields don't differ substantially from the annual distribution.

Let us consider the winter analysis (fig. 2). The intensities of the divergence and convergence centers are, in general, much higher and the convergence extends deeper to lower latitudes. In fact, two large zonal belts of convergence alternating with zones of divergence may be observed. In middle latitudes the transition from divergence to convergence regions, $\operatorname{div} \overline{\vec{S}}' = 0$, is now much better defined. Australia, South Africa, and most of South America are under the influence of a much more intense divergence than it is revealed by the annual map. Furthermore, the isolated centers of convergence shown in the yearly analyses almost disappeared. The centers of divergence over the South Pacific, over the Indian and over the Atlantic Oceans are more intense in the winter than in the year. However, relatively to the continents they are now much less intense.

The analysis relative to the summer semester (fig. 3) shows the same general features, but the intensities of the centers are much weaker than in the preceding cases, as was to be expected. The intensities of the field over the oceans and over the continents are now reversed as regards to the winter distribution. Now the divergence is slightly higher over the oceans. The influence of the continents is very striking when the seasonal distribution over Australia, South America and Africa are compared. Although there is divergence in summer and winter, the values of the latter are much more intense. This is, however, the general rule, even for the convergence, as can be seen in comparing both distributions in the high latitude belt of convergence.

In the study of the energetic balance of an oceanic region and in the global study of the flux of energy, it cannot be forgotten the important role played by oceans in the storing of enthalpy which is much higher than that of the atmosphere and in its distribution and transport effected as well as by surface currents, as by internal currents of oceans. The flux of energy is considerable and can reach relatively high values.

The analysis demonstrate two other features of this eddy heating. Firstly note the area of intense heating in polar regions during the winter months. This is to be expected since during the summer, the polar regions are receiving much more solar radiation than in the winter months when they are in continual darkness. It should also be noticed that the relative areas of heating and cooling display more seasonal dependence in midlle to high latitudes than in the tropics. This is also commensurate with radiation results.

From the inspection of the maps, it is seen there, that there is a net heating in the equatorial regions. This is not, however, as questionable a result as might be thought. Equatorial heat transports in the high equatorial troposphere have been noted by Peixoto (1960a), Starr and Wallace (1964), and Gilman (1964). In view of this the actual results seem quite plausible.

The existence, in the mean, of an inflow of enthalpy through the equator leads to the existence of an excess in the energy balance of northern hemisphere. This result agrees with the values obtained by Budyko according to which northern hemisphere receives in the mean more energy, i. e. 479 cal/(cm^2. year) as compared with 467 cal/(cm^2. year) received by the southern hemisphere.

4. 2. *Vertical distribution of the mean meridional divergence of enthalpy*

Vertical distributions of the zonally averaged values of the divergence of total eddy flux of enthalpy (transient plus standing eddy) at various isobaric levels are presented in figs. 4, 5 and 6. The values

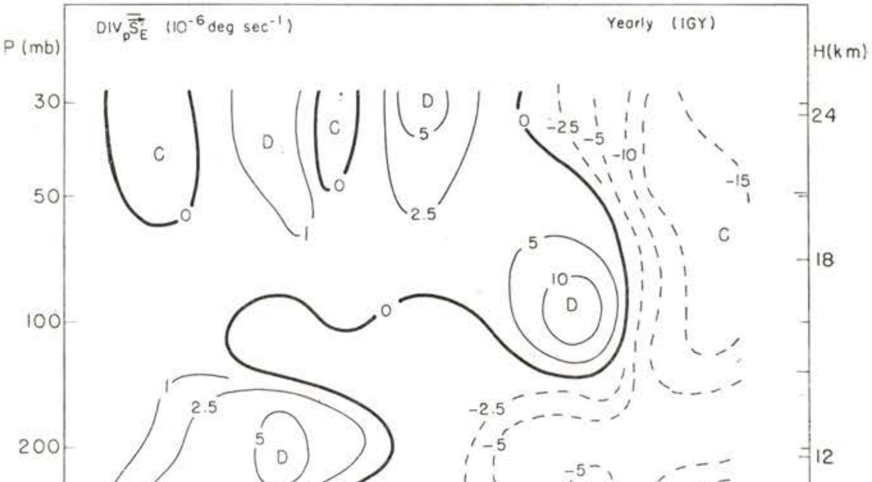

Fig. 4 — Vertical meridional cross-section through the atmosphere showing the distribution of the mean meridional eddy divergence of enthalpy for yearly data. The units are 10^{-6} deg sec^{-1}.

were computed from the mean total eddy fluxes at various levels ($\vec{\bar{\bar{S}}}_E = \vec{\bar{\bar{S}}}' + \vec{\bar{\bar{S}}}^*$) published in a previous paper (Peixoto, 1974).

The values of $\operatorname{div} \vec{\bar{\bar{S}}}_E = \operatorname{div} \vec{S}' + \operatorname{div} \vec{\bar{\bar{S}}}^*$ were computed for the zone bounded by latitude Φ_1 and Φ_2 through the variant of expression (17)

$$\operatorname{div} \vec{\bar{S}}_E = \frac{1}{2\pi(\sin\Phi_2 - \sin\Phi_1)} \oint \{[\bar{S}_{E,\Phi} \cos\Phi]_{\Phi_2} - \\ - [S_{E,\Phi} \cos\Phi]_{\Phi_1}\} \, d\lambda . \tag{18}$$

These cross-sections give the main characteristics of the distribution of the divergence of enthalpy along the vertical. By and large,

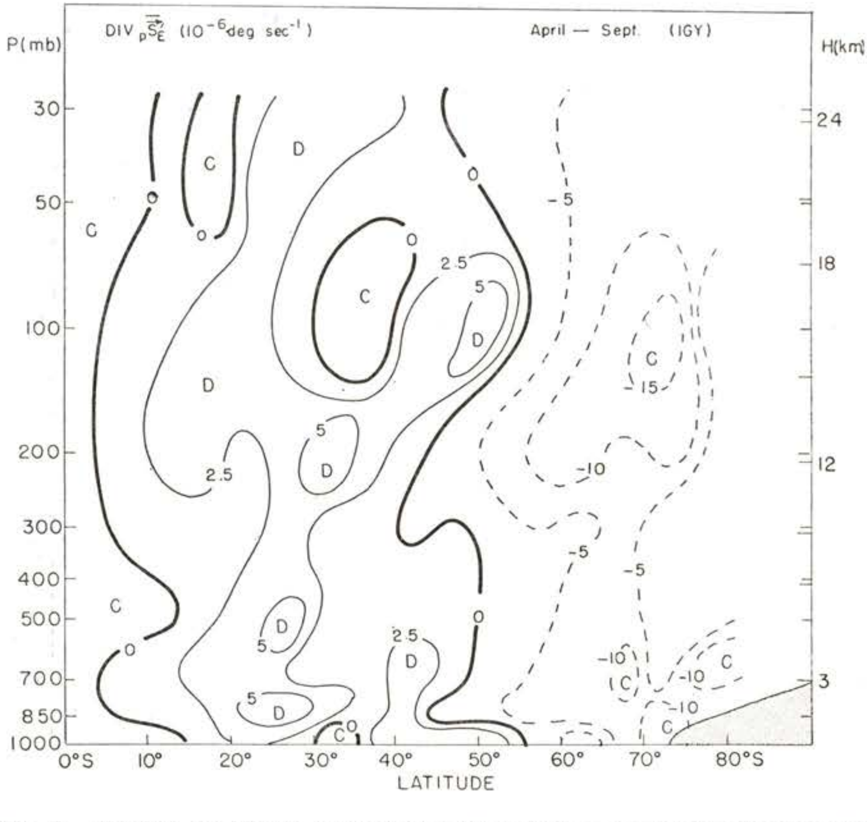

Fig. 5 — Vertical meridional cross-section through the atmosphere showing the distribution of the mean meridional eddy divergence of enthalpy for winter data. The units are 10^{-6} deg sec^{-1}.

divergence predominates north of 50° S at all levels with two main centers in the lower and upper troposphere. South of 50° S, convergence prevails, and is much more intense in the stratosphere. In the equatorial region there is also a slight convergence. In winter, both the divergence and convergence centers are more intense in the

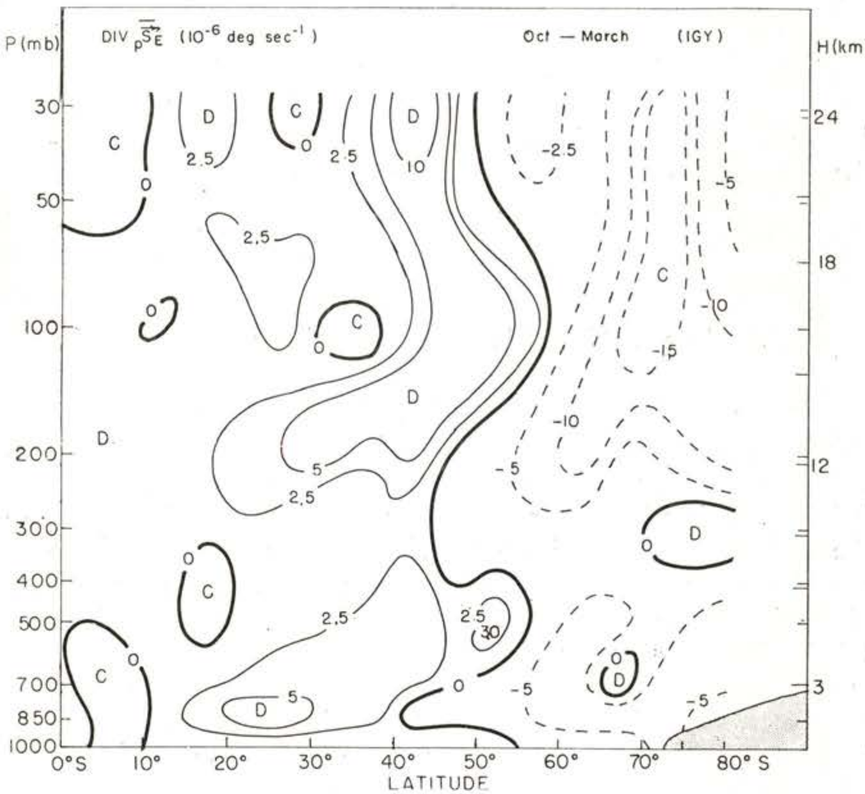

Fig. 6 — Vertical meridional cross-section through the atmosphere showing the distribution of the mean meridional eddy divergence of enthalpy for summer data. The units are 10^{-6} deg sec^{-1}.

troposphere than they are in summer. The intense convergence observed in the high to subpolar latitudes at all levels in the stratosphere during the winter are associated with the cooling caused by polar night. The centers in the lower troposphere are associated with the transient perturbations along the polar front and with the semi-permanent lows which prevail in high latitude levels.

On the other hand the main divergence centers observed in the upper levels are associated with the warming due to the net radiation excess, solar minus terrestrial, observed in the low latitudes whereas those observed in the lower troposphere are associated with the semi-permanent anticyclones belt centered over the subtropical latitudes

The centers of divergence and of convergence in the lower troposphere suggest that the effects of winter cooling and summer warming due to the presence of the globe do not penetrate much above 500 mb. The similarity of the two seasonal distributions in the lower layers must be due to the existence of large oceanic expanses in the Southern Hemisphere. However, longitudinal cross-sections, for example, along the latitude circle of 20° S, would show for both seasons significant differences over the continents as compared with the oceans.

4. 3. *Mean meridional divergence of enthalpy*

In the studies of the global heat budget of the earth or of the subsystems globe and atmosphere, it is convenient to use the mean meridional profiles of the various components of the budget at given latitudes. This procedure gives a general view of the importance of the various components in achieving the required energy balance and provides some insight on the mechanisms which run the general circulation of the atmosphere. Moreover, it makes the analysis of the relative influence of the various forms of energy in the global process easier.

Since the enthalpy is one of the main components of the heat budget through its divergence or convergence, the corresponding vertically integrated values of the mean meridional divergence of the transient and stationary modes, as well as the total eddy divergence up to 50 mb, was evaluated at various latitudes. The values for yearly, winter and summer seasons are presented in Table 1 in units of watt/m^2 and in degree/day. The corresponding meridional profiles with the latitudinal distributions of the various modes of divergence are also shown in Fig. 7. The abscissa axis is graduated in degrees of latitude proportional to $\sin \Phi$, so that areas bounded by the profile curves are comparable with each other.

The results were obtained using the values of the total mean stationary and transient eddy transports already published (Peixoto, 1974) applying the formulation expressed by (18). The convergence

TABLE 1. *Zonally averaged divergence of the total eddy flux of enthalpy of the atmosphere in the Southern Hemisphere in units of watt/m² and in degree/day between brackets* (1 watt $m^{-2} = 1.435 \times 10^{-3}$ cal cm^{-2} $min^{-1} = 0{,}754$ kilolangleys $year^{-1}$).

Lat (S)	Annual			Winter			Summer		
	$\overline{\mathrm{div}\ \vec{S}'}$	$\overline{\mathrm{div}\ \vec{S}^*}$	$\overline{\mathrm{div}\ \vec{S}_E}$	$\overline{\mathrm{div}\ \vec{S}'}$	$\overline{\mathrm{div}\ \vec{S}^*}$	$\overline{\mathrm{div}\ \vec{S}_E}$	$\overline{\mathrm{div}\ \vec{S}'}$	$\overline{\mathrm{div}\ \vec{S}^*}$	$\overline{\mathrm{div}\ \vec{S}_E}$
0–10°	14,84	13,06	27,90	−5,93	19,00	13,06	0,59	27,32	27,91
	(0,13)	(0,15)	(0,24)	(−5,27)	(0,16)	(0,11)	(52,71)	(0,24)	(0,24)
10–20	77,79	−4,90	72,90	52,07	29,40	81,47	53,90	−13,47	40,43
	(0,68)	(−4,34)	(0,65)	(0,46)	(0,26)	(0,72)	(0,47)	(−0,11)	(0,35)
20–30	168,44	16,32	184,80	190,64	17,62	208,27	146,89	8,48	155,38
	(1,49)	(0,41)	(1,63)	(1,69)	(0,15)	(1,84)	(1,30)	(7,53)	(1,37)
30–40	161,80	−35,40	126,41	251,62	−104,74	110,51	164,69	−38,28	126,41
	(1,43)	(−0,31)	(1,12)	(1,91)	(−0,92)	(0,98)	(1,46)	(−0,33)	(1,12)
40–50	24,26	2,51	26,78	46,86	51,04	97,90	66,94	20,28	87,02
	(0,12)	(2,22)	(0,23)	(0,41)	(0,45)	(0,86)	(0,59)	(0,17)	(0,77)
50–60	244,49	57,77	186,72	−267,19	53,64	−213,54	−202,19	63,96	138,23
	(2,17)	(0,51)	(−1,65)	(−2,37)	(0,47)	(−1,89)	(−1,79)	(0,56)	(−1,22)
60–70	385,03	37,80	347,72	−441,03	−9,80	−450,83	−358,42	36,40	−322,02
	(3,41)	(0,33)	(−3,08)	(−3,91)	(−8,69)	(−4,00)	(−3,18)	(0,32)	(−2,85)
70–80	267,48	−116,59	384,04	−260,62	−86,87	−347,50	−210,33	−93,73	−304,46
	(2,37)	(−1,03)	(−3,40)	(−0,23)	(−0,77)	(−3,08)	(−1,86)	(−0,83)	(−2,69)

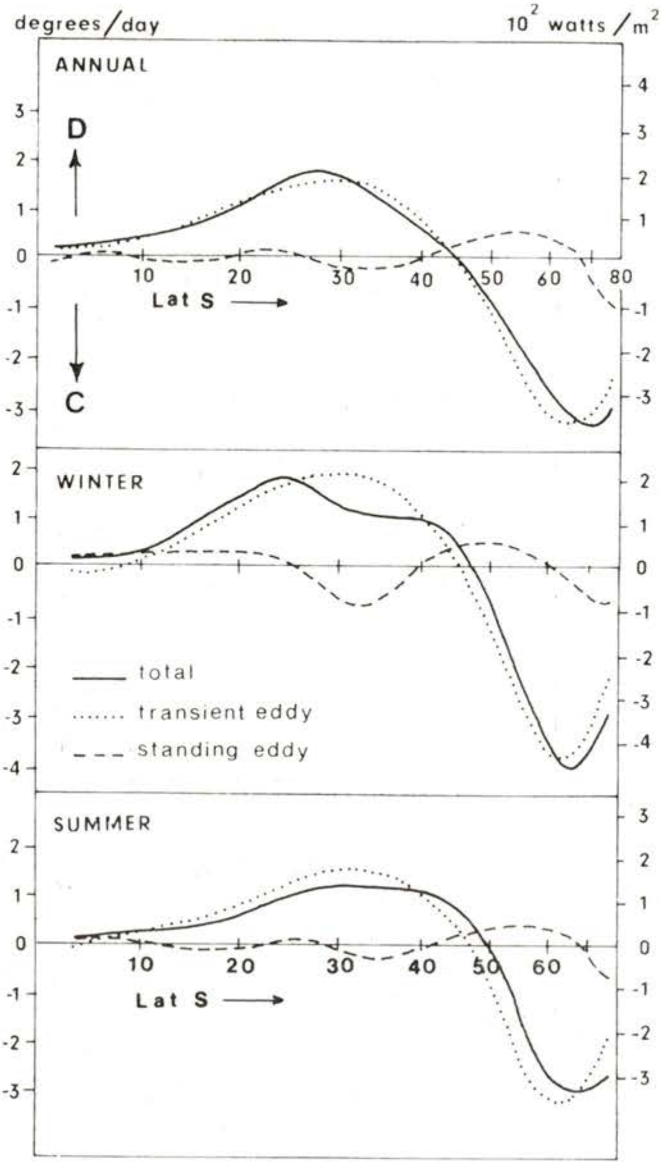

Fig. 7 — Profile of the mean meridional divergence of total eddy flux of enthalpy in units of cal/(cm². min), watt m² and °K/day, for yearly, winter and summer data.

or divergence of enthalpy was also evaluated in degrees per day of uniform warming or cooling of the atmosphere within a ten-degree latitude zone, for a depth of the atmosphere from 1000 to 50 mb.

The mean zonal values of the divergence of transient eddy flux of enthalpy, $[\overline{\mathrm{div}\,\vec{S}'}]$, constitute a synthesis of the maps previously discussed. The mean annual profile shows that there is divergence in low and middle latitudes, $(\Phi \leq 45^{\circ}\,S)$, and convergence polewards of 45° S and near the equator, although slight. The divergence predominates over the tropical to middle latitudes regions.

The marked convergence of enthalpy observed in high and subpolar latitudes with an extreme of $-0.58 \times 10^{+3}$ watt/m^2 ($+3{,}4$ degrees/day) near 65° S, is associated with the flux due to the transient perturbations predominating in those regions. This convergence is going to compensate the intense cooling due to the radiative deficit observed in the subpolar latitudes.

The weak convergence observed in the vicinity of the equator is mainly due to the perturbations along the intertropical convergence zone (ITCZ) and to its latitudinal shift during the year.

The divergence observed over the tropics and middle latitudes is generated by the heating due to the surplus of the radiation existing in this extense region. It presents a maximum of the order of $0{,}17 \times 10^{3}$ watt/m^2 which is equivalent to a cooling rate of 1,5 degree/day. This export of enthalpy by the eddy circulations of the atmosphere plays a decisive role in the overall balance of heat and energy of the earth system as a whole, as well as of the atmosphere and of the oceans, since it prevents a continuous accumulation of heat in these regions.

The profiles relative to the divergence of stationary eddy fluxes $\overline{\mathrm{div}\,\vec{S}^{*}}$ are in general much less intense than those due to the transient eddy transport. The values of $\overline{\mathrm{div}\,\vec{S}^{*}}$ show alternating zones of divergence and of convergence. As the profiles show, there is slight convergence in the tropical, middle and subpolar latitudes and mean divergence in the high latitudes. It is noteworthy to point out the high values of the divergence in the high latitudes and the systematic convergence observed in the subpolar regions. As expected, these features are associated with the fluxes due to the predominance of the semi-permanent perturbations of the general circulation of the atmosphere, namely the tropical anticyclones and the semi-permanent lows in the high latitudes.

The profiles of the total eddy divergence $\overline{\mathrm{div}\,\vec{S}_E}$ combined the divergence of both eddy components, as can be seen from the inspection of values of Table 1 and from the meridional curves in figures 7, 8 and 9.

In analysing the winter and summer values and in comparing them with the corresponding annual results, if we discount the larger values in the winter season, it is apparent the similarity of all the distribution. This is due to the homogeneity of the interface conditions in the Southern Hemisphere.

A comparison of the results just discussed, with those for the Northern Hemisphere, (Peixoto, 1967), shows by and large a good similarity. The Southern Hemisphere values are, as a rule, smaller than those in the Northern Hemisphere. This is particularly true when one considers the divergence of stationary eddy flux, which must be related to the difference in the geography, since the standing eddies are affected directly by the surface inhomogeneities.

As regards to the interannual and seasonal variations of the various modes of divergence of enthalpy, they are much less pronounced in the Southern Hemisphere than in the Northern Hemisphere, as expected.

5 — FINAL COMMENTS

a) In this study not all the modes of divergence of enthalpy were considered, namely those associated with the mean meridional circulations ($\mathrm{div}\,\overline{\vec{S}_M}$). For the toroidal circulation the value of $\overline{S}_{M\Phi}$ is given by

$$\overline{S}_{M\Phi} = \frac{C_p}{g} \int [\bar{v}]\,[\overline{T}]\,dp$$

noting that by continuity considerations in the long term $\int [\bar{v}]\,dp = 0$.

The computation of $\overline{S}_{M\Phi}$ involves the evaluation of the mean meridional component of the wind $[\bar{v}]$ from the actual observations and the estimates of $[\bar{v}]$ involve a very high degree of uncertainty (the geostrophic winds at any isobaric level are such that: $[\bar{v}_{gs}] = 0$). It is expected that the divergence of enthalpy by the mean meridional circulations becomes important in the tropics, mainly due to the effect of the Hadley cell.

It must be pointed out, however, that this form of divergence of enthalpy should not be taken independently from the equivalent divergence of potential energy because the meridional transports of both forms of energy by the toroidal circulations are intimately connected. In fact, the toroidal circulations are efficient in transporting enthalpy in the tropics toward the equator in the low levels and potential energy away from the equator in the upper levels leading to a relative net cooling near the equator and heating in the subtropical latitudes. Furthermore, it is well known that in an adiabatic atmosphere the mean meridional flux of enthalpy should be compensated exactly by a flux of potential energy in the opposite sense $[\overline{g\,dz + C_p\,dT}][\bar{v}] = 0$.

This point deserves further study.

b) It should be noted that in the present study only one part of the total flux of the global energy was treated, namely that associated with the enthalpy transport. In evaluating the heat balance by the dynamic method through the divergence of total energy fluxes as was mentioned, the enthalpy and the potential energy $(\Phi = g\,z)$ transports must be taken together, simultaneously. The divergence of the flux of the kinetic energy can be neglected since it is very small when compared with the divergence of the other mentioned forms of energy.

Nevertheless the actual results already point out in the right direction. The heating or the cooling due to the convergence or to the divergence of enthalpy act in a sense opposite to the radiative heating as required by previous radiative studies (London, 1971). This is particularly true in the subtropical and intermediate latitude regions, where most of the divergence is due to the presence of transient eddies. It so happens that for this mechanism the influence of the potential energy is negligible, since the time and space covariances of T and v are much larger than those of Z and v, as it is known from synoptic experience. Therefore the divergence of the eddy flux of potential energy does not count for the heat balance. In the low and polar latitudes the situation may become somewhat different.

c) With the IGY homogeneous sets of data it was possible to elaborate a consistent study of the various forms of energy transports in the Southern Hemisphere. It is then possible to assess now the balance of energy, on a global scale, by the dynamic method, using the fluxes of potential energy, enthalpy and latent heat, combined with the radiation data. This results from the consideration of condition (5) when applied to equation (1) in a steady regime. From scale analysis considerations it is obvious that $\{\dot{Q}_F\}$ can be disregarded in comparison with the others terms. This shows that the high latitude heating by flux convergence is compensated by net loss of heat, and the cooling in low latitudes by net gain of heat by the atmosphere through the following processes: (1) convergence, or divergence, of enthalpy flux in the atmosphere maintained by mean meridional or eddy circulations of permanent or intermitent kind, (2) net gain, or loss, of heat by radiation, (3) net gain, or loss, by conduction of sensible heat through the interface with the ocean and the solid ground, and (4) net gain of heat of condensation by the precipitation of rain or snow.

d) In discussing the energy balance of the earth, the oceans and the atmosphere must be taken as a single system, since both carry and store energy. The vectorial field $\vec{S}(\lambda, \Phi)$ must then be formed by the superposition of the oceanic and atmospheric transport fields $\vec{S}_0(\lambda, \Phi)$ and $\vec{S}_a(\lambda, \Phi)$, respectively:

$$\vec{S}(\lambda, \Phi) = \vec{S}_a(\lambda, \Phi) + \vec{S}_0(\lambda, \Phi).$$

Separate budgets for each subsystem, the atmosphere, or the oceans, can be determined when the exchange of energy across its interface is known.

The enthalpy balance for a polar cap south of $\Phi°$ S which includes the atmosphere, the oceans and cryosphere can then be written symbolically, assuming that the change of energy storage can be disregarded as follows:

$$\operatorname{div} \vec{S}_a + \operatorname{div} \vec{S}_0 = \{\dot{Q}_R\} + \{\dot{Q}_L\} + \{\dot{Q}_C\}.$$

The values of the atmospheric flux $\vec{S}_a$ can be estimated rather accurately from the radiosonde network. On the contrary the transports by ocean currents $\vec{S}_0$ are not well known, since the available estimates are obtained indirectly.

Reliable estimates of the radiation flux $\{\dot{Q}_R\}$ have recently become available from satellite data (Von der Haar and Suomi, 1971).

The values of $\{\dot{Q}_L\}$ can also be estimated from aerological data, since for the system oceans-atmosphere

$$\{\dot{Q}_L\} = \{L \operatorname{div} \vec{Q}\} = L(E - P).$$

Where $\vec{Q}$ is the water vapour transport in the atmosphere, L the latent heat, E the evaporation and P the precipitation (Peixoto, 1972).

For this system $\{\dot{Q}_C\}$ need not to be considered. Thus, one obvious method of estimating the ocean enthalpy flux, is from previous equation as a residual. Robinson (1970) in a preliminary study of the heat flux by the oceans in the Southern Hemisphere came to the conclusion that it seems to be of the same order as the values of the atmospheric energy flux.

This point requires further study and should be investigated using a more homogeneous set of data now available for the Southern Hemisphere. Using the dynamic approach it will be possible to infer the role played by the oceans in the energy and heat balances and to compare them with previous estimates (Sellers, 1965; Emig 1967, Newton, 1972, etc.).

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