INTERFERENCE EFFECTS IN HEAVY ION ELASTIC AND INELASTIC SCATTERING (*)

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ABSTRACT — The semi-classical theory is used to describe the different oscillatory behaviours observed in the elastic and inelastic heavy ion scattering cross-sections at incident energies near and above the Coulomb barrier. The theory predicts two different phases rules which are well observed in the analyzed data: elastic and inelastic scattering cross-sections of ^{11}B on ^{208}Pb at EL = 72.2 MeV and of ^{12}C on ^{27}Al at EL = 46.5 MeV.

1 — INTRODUCTION

In heavy ion reactions, the structure of the elastic and inelastic scattering cross-sections strongly depends on the scattering angle region and on the incident energy. At energies below or just near the Coulomb barrier, the nuclear potential can be treated as a small perturbation to the Coulomb one and the classical deflection function is nearly of Coulomb form. The agreement of semi-classical theory to describe such process is well known [1]-[5] [17].

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At higher energies [6]-[9] (above the Coulomb barrier), there exist a scattering angle θ_r close to the angle of grazing collision for which the deflection function is stationary. On one side $\theta > \theta_r$ the elastic and inelastic scattering cross-sections drop rapidly to very low values, on the other side $\theta < \theta_r$ they present «out of phase» oscillations irrespective of the parity of the transition [14] [16].

In the lit region $\theta < \theta_r$ the observed phase rule results from the quantal interference effect between two classical trajectories which contribute to the cross-sections at each scattering angle, and to the fact that the one, with high impact parameter is mainly described by the Coulomb forces whereas the other, with low impact parameter is mainly described by the nuclear forces. The contribution of the negative branch of the deflection function can be neglected (Section 2. 2).

In the dark region $\theta > \theta_r$ the scattering is essentially described by the rainbow process and drops exponentially to very low values. When the incident energy increases [15], the elastic and inelastic cross-sections present in the lit region the previously mentioned «out of phase» oscillations structure. On the dark side of θ_r, the fall-off is corrected by oscillations. This oscillations structure observed for $\theta > \theta_r$ can be interpreted as the interference between the rainbow scattering (decreasing fastly) and that defined by the negative branch of the classical deflection function whose relative contribution increases with the incident energy (Section 2. 3). In the inelastic cross-sections, the theory predicts oscillations which are «in phase», or «out of phase» with the elastic one according as the transition is even or odd. This result his exactly the inverse of the well-known Blair phase rule [26].

The forms of the ion-ion potentials and the resulting classical deflection functions are discussed in Section 3. The theory outlined in Section 2 is used to describe:

- the elastic scattering and inelastic 3^- (2.61 MeV), 5^- (3.20 MeV), 2^+ (4.10 MeV) and 4^+ (4.31 MeV) transitions in ^{208}Pb in the collision with ^{11}B at laboratory incident energy of 72.2 MeV
- the elastic scattering and inelastic 2^+ (4.43 MeV) transition in ^{12}C in the collision of ^{12}C on ^{27}Al at laboratory energy of 46.5 MeV.

The experimental data [9] [15] and the theoretical results are presented in Section 4.

2 — INELASTIC SCATTERING

2.1. *Outline of the theory*

The semi-classical formulation of the inelastic cross-section is developed by use of mathematical approximations analogous to those defined in the description of the elastic scattering [10]-[13].

In order to discuss correctly the approach to the classical limit, it is most expedient to start from the scattering amplitude in the form usually defined in quantum theory for a central interaction [16].

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} \frac{2 I_f + 1}{2 I_i + 1} \sum_{LM} \delta(l, I_i, I_f) \frac{|\beta_{LM}|^2}{2L+1} \qquad (2.1)$$

$$\beta_{LM} = (4\pi)^2 \sum_s \sum_{l_i m_i} \sum_{l_f m_f} i^{l_i + l_f - l}\, Y^*_{l_i m_i}(\hat{k}_i)\, Y^*_{l_f m_f}(-\hat{k}_f)$$

$$\exp(i(\eta_{l_i} + \eta_{l_f})) < l_i m_i l_f m_f | L M > < l_i \circ l_f \circ | L \circ >$$

$$\frac{\hat{l}_i \hat{l}_f}{(4\pi)^{1/2} \hat{L}} A^s_L I^s_{l_i l_f}(L) \qquad (2.2)$$

where

$$I^s_{l_i l_f}(L) = \frac{1}{k_i k_f} \int_0^\infty dr\, u_{l_i}(k_i r)\, u_{l_f}(k_f r)\, F^s_L(r). \qquad (2.3)$$

For the incoming waves $u_{l_i, l_f}(kr)$, we assume the form defined outside the nuclear region

$$u_l^-(kr) = \cos \delta_l\, F_l(kr) + \sin \delta_l\, G_l^-(kr) \qquad (2.4)$$

where F_l, G_l are the regular and irregular Coulomb wave functions and δ_l is the nuclear phase shift of the l-partial wave.

In (2.2) and (2.3), L defines the multipolarity of the transition, A^s_L and F^s_L are respectively the spectroscopic factor and the form factor of the Coulomb or nuclear excitation forces used to describe the interaction of the ions. Their explicit forms are defined in the framework of a collective model (vibrational or rotational) of the target nucleus [1] [16]. In (2. 1), (2. 2), (2. 3), μ_{if}, k_{if}, η_{if} are respectively the reduced masses, the wave numbers and the total (Coulomb + nuclear) phase shifts in the incomming (i) and outgoing (f) channels.

In near classical conditions, many partial waves contribute to the sum in (2. 2) and we may introduce some approximations. We take the variables (L, M) fixed and let the angular momentum l_i, l_f become infinite with the difference $\mu = l_f - l_i$ finite. The classical limit of the radial integrals (2. 3) have already been discussed [17]

$$I^s_{l_i, l_f}(L) \sim \frac{1}{2k^2} a \cos(\delta_{l_i} - \delta_{l_f}) \int_{-\infty}^{+\infty} d w \exp(i \xi (\varepsilon \sin h w + w))$$

$$\frac{(\varepsilon + \cos h w + i \sin h w (\varepsilon^2 - 1)^{1/2})^{\mu}}{(\varepsilon \cos h w + 1)^{\mu - 1}} F^s_L(r(w))$$

$$= I^s(l, \mu, L) . \qquad (2.\ 5)$$

The use of the variable «w» is the usual parametrization defined for the radial variable in semi-classical Coulomb theory

$$r = a(\varepsilon \cos h w + 1) \qquad (2.\ 6)$$

where $a = \eta^c / k$ for the Coulomb trajectory.

η^c is the Sommerfeld number and

$$\varepsilon = \frac{(\eta_c^2 + (l + 1/2)^2)^{1/2}}{\eta^c} . \qquad (2.\ 7)$$

In order to improve the semi-classical results, we use some average values between the initial and final ones for η^c, k and l:

$$\eta^c = (\eta^c_i \, \eta^c_f)^{1/2} \quad k = (k_i k_f)^{1/2} \quad l = \frac{l_i + l_f}{2} \qquad (2.\ 8)$$

To proceed further, we replace the sums $\sum_{l_i} \sum_{l_f}$ by $\sum_{\mu} \int_0^{\infty} d l$ and we require the asymptotic forms for the spherical harmonics, the total phase shift and the Wigner coefficients [18]

$$Y_{lm}(\theta, \varphi) \sim \frac{1}{\pi} e^{i m \varphi} \frac{1}{(\sin\theta)^{1/2}} \cos\left((l + 1/2)\theta + (m - 1/2)\frac{\pi}{2}\right)$$

valid for

$$\frac{1}{l} \leq \theta \leq \pi - \frac{1}{l}$$

$$\eta_{l_i, l_f} \sim \overset{\text{W. K. B}}{\eta(l)} = \frac{1}{\hbar}\left(\frac{\pi}{2}\hbar(l+1/2) - \int_{r_m}^{\infty} d r\, r\, d_r\, F^{1/2}(r)\right)$$

where

$$F(r) = 2m\left(E - v(r) - \frac{\hbar^2 (l+1/2)^2}{r^2}\right)$$

$$< l_i o l_f M \mid L M > \sim (-)^{l_i + L + M}\left(\frac{2L+1}{2 l_f + 1}\right)^{1/2} D^{L}_{l_f - l_i, M}\left(0, \frac{\pi}{2}, 0\right). \quad (2.9)$$

The classical expression of the scattering coefficient (2. 2) is:

$$\beta^c_{LM} = 4 \sum_s \sum_\mu \frac{(4\pi)^{1/2}}{2L+1} \frac{i^{-L}}{(\sin\theta)^{1/2}} Y_{L\mu}\left(\frac{\pi}{2}, 0\right) d^L_{M,-\mu}(\pi/2)$$
$$e^{-iM(\varphi+\pi)} e^{i\mu\frac{\pi}{2}} A^s_L (I^+_{L\mu s} + I^-_{L\mu s}) \quad (2.10)$$

where

$$I^{\pm}_{L\mu s} = \frac{1}{\sqrt{2}} e^{\pm i \frac{\pi}{4}} e^{\mp i M \frac{\pi}{2}} \int d l\, \Gamma^s(L, \mu, l)\, l^{1/2} e^{i(2\eta(l) \mp l\theta)} \quad (2.11)$$

To obtain an accurate evaluation of the semi-classical expression (2. 10) we must examinate the various approximations available to evaluate the integrals $I^{\pm}_{L\mu s}$ (2. 11). The classical deflection functions $\Theta(l)$ we consider to describe heavy ions collision will be explicitly defined in Section 3.

2. 2. *Analysis in the lit region of* θ_r

In the lit region $\theta < \theta_r$, the most important contributions to the scattering come from Coulomb and nuclear surface trajectories defined by $\theta = +\Theta(l)$. So, in (2, 10), we only retain the $I^+_{L\mu s}$ terms. In order to calculate the amplitude integral (2. 11), let us define the new variable [12] :

$$\varphi^{+}(l) = 2\,\eta\,(l) - l\,\theta = \xi\,(\theta)\,x + \frac{x^3}{3} + A\,(\theta) \qquad (2.\ 12)$$

such that

$$I^{+}_{L\,\mu\,s} = e^{-iM\frac{\pi}{2} + i\frac{\pi}{4}}\; e^{i\,A(\theta)}$$

$$\frac{1}{\sqrt{2}} \int_0^{\infty} \lambda^{1/2}(x)\,\frac{d\,\lambda(x)}{d\,x}\, I^{s}(L\,,\mu\,,\lambda(x))\, e^{i(\xi(\theta)x + x^3/3)}\,d\,x\,. \qquad (2.\ 13)$$

Only the stationary points regions at $x = \pm\, i\,\xi^{1/2}(\theta)$ are considered to contribute to the integral, and we stipulate the correspondence to be

$$\begin{aligned} l = l_1 &\longleftrightarrow x = -\,i\,\xi^{1/2}(\theta) \\ l = l_2 &\longleftrightarrow x = +\,i\,\xi^{1/2}(\theta) \end{aligned} \qquad (2.\ 14)$$

When alternatively inserted in (2. 12), these relations yield the values of $A(\theta)$ and $\xi(\theta)$. For $\theta < \theta_r$, l_1 and l_2 are real values and $\xi(\theta)$ will be real and negative : $\xi(\theta) = -\,|\,\xi(\theta)\,|$. In order to evaluate (2. 13) we introduce the serie :

$$\lambda^{1/2}(x)\,\frac{d\,\lambda(x)}{d\,x}\, I^{s}(L\,\mu\,\lambda(x)) = \sum_{m=0}^{\infty} (x^2 + \xi(\theta))^m\,(p_m + q_m\,x)\,. \qquad (2.\ 15)$$

The stationary phase approximation in (2. 13) defined for $\xi(\theta) + x^2 = 0$, corresponds to the lowest order $m = 0$ approximation (2. 15). Using the relations (2. 14), p_0 and q_0 are defined by

$$p_0 = \frac{1}{2}\,(i_1 + i_2) \qquad q_0 = \frac{1}{2\,i\,\xi^{1/2}}\,(i_2 - i_1) \qquad (2.\ 16)$$

with

$$i_{\substack{1\\2}} = \lambda_{\substack{1\\2}}^{1/2} \left[\frac{2\,i\,\xi^{1/2}}{|\,\Theta'(l)\,|_{l_{1,2}}} \right]^{1/2} I^{s}(L\,\mu\,\lambda_{\substack{1\\2}})\,. \qquad (2.\ 17)$$

In these expressions (2. 17) the derivative is supposed to be negative for $l = l_1$ and positive for $l = l_2$ (Fig. 1-2). By inserting (2. 15)-

(2. 17) into (2. 13) and integrating, the inelastic scattering integral is approximated by:

$$\mathrm{I}^{+}_{\mathrm{L}\,\mu\,s} = \pi\, e^{-i\mathrm{M}\frac{\pi}{2} + i\frac{\pi}{4}}\, e^{i\mathrm{A}(0)}\, .$$

$$\Big\{(-\xi)^{1/4}\,\mathrm{A}i(\xi)\Big[\Big(\frac{l_1+1/2}{|\Theta'(l)|_{l_1}}\Big)^{1/2}\mathrm{I}^{s}(\mathrm{L}\,\mu\,l_1) + \Big(\frac{l_2+1/2}{|\Theta'(l)|_{l_2}}\Big)^{1/2}\mathrm{I}^{s}(\mathrm{L}\,\mu\,l_2)\Big]$$

$$-\frac{i}{(-\xi)^{1/4}}\,\mathrm{A}'i(\xi)\Big[\Big(\frac{l_2+1/2}{|\Theta'(l)|_{l_2}}\Big)^{1/2}\mathrm{I}^{s}_{(\mathrm{L}\,\mu\,l_2)} - \Big(\frac{l_1+1/2}{|\Theta'(l)|_{l_1}}\Big)^{1/2}\mathrm{I}^{s}_{(\mathrm{L}\,\mu\,l_1)}\Big]\Big\} \qquad (2.\ 18)$$

It we describe the excitation of a vibrational even-even nucleus from its fondamental 0^{+} to a state $\mathrm{I}^{(-)^{\mathrm{I}}}$, the expression for the cross-sections is:

$$\frac{d\sigma^{0,\mathrm{I}}}{d\Omega} = \eta^{c^2}\,\pi\,(1-\tau)^{1/2}\sum_{\mu}\Big|\mathcal{Y}_{\mathrm{I}\mu}\Big(\frac{\pi}{2},0\Big)\Big|^{2}\cdot$$

$$|\xi|^{1/2}\,\mathrm{A}^{2}i(-|\xi|)\cdot[\sigma^{c}_{(1)}\,|a_{0\mathrm{I},\mu}(l_1)|^{2} + \sigma^{c}_{(2)}\,|a_{0\mathrm{I},\mu}(l_2)|^{2}$$

$$+(\sigma^{c}_{(1)}\,\sigma^{c}_{(2)})^{1/2}(a_{0\mathrm{I},\mu}(l_1)\,a^{*}_{0\mathrm{I},\mu}(l_2) + a^{*}_{0\mathrm{I},\mu}(l_1)\,a_{0\mathrm{I},\mu}(l_2))]$$

$$+\frac{1}{|\xi|^{1/2}}\,\mathrm{A}'^{2}i(-|\xi|)\cdot[\sigma^{c}_{(1)}\,|a_{0\mathrm{I},\mu}(l_1)|^{2} + \sigma^{c}_{(2)}\,|a_{0\mathrm{I},\mu}(l_2)|^{2}$$

$$-(\sigma^{e}_{(1)}\,\sigma^{c}_{(2)})^{1/2}(a_{0\mathrm{I},\mu}(l_1)\,a^{*}_{0\mathrm{I},\mu}(l_2) + a^{*}_{0\mathrm{I},\mu}(l_1)\,a_{0\mathrm{I},\mu}(l_2)]$$

$$+2\,i\,\mathrm{A}i(-|\xi|)\,\mathrm{A}'i(-|\xi|)\,(\sigma^{c}_{(1)}\,\sigma^{c}_{(2)})^{1/2}$$

$$\cdot[a_{0\mathrm{I},\mu}(l_1)\,a^{*}_{0\mathrm{I},\mu}(l_2) - a^{*}_{0\mathrm{I},\mu}(l_1)\,a_{0\mathrm{I},\mu}(l_2)]\} \qquad (2.\ 19)$$

with

$$|\xi| = \Big[\frac{3}{4}\,\partial(\theta)\Big]^{2/3} \qquad (2.\ 20)$$

$\sigma^{c}_{(i)}$ and $\partial(\theta)$ are respectively the classical elastic cross-section and the phase difference [10]-[13]

$$\sigma^{c}_{(i)} = \frac{l_i + 1/2}{k^{2}\sin\theta\,|\Theta'(l)|_{l_i}} \qquad (2.\ 21)$$

$$\delta(\theta) = \int_{l_2(\theta)}^{l_1(\theta)} |(\Theta)(l) - \theta| \, dl = 2\eta(l_1) - l_1\theta - 2\eta(l_2) + l_2\theta . \qquad (2.\ 22)$$

The coefficient $(1-\tau)^{1/2}$ where $\tau = \dfrac{\Delta E_{0,I}(A_T + A_P)}{A_T \cdot E_L}$ results from the use of the symmetrized form (2. 8) for the impulse parameter k.

The inelastic amplitudes $a_{0I,\mu}(l_i)$ have been explicitly defined in references [1] [16].

Far from $\theta_r (\theta << \theta_r), |\xi(\theta)|$ is large; the Airy functions and their derivatives may be replaced by their asymptotic expressions [19] and (2. 19) reduces to the form

$$\frac{d\sigma^{0I}}{d\Omega} = \eta^{c^2} (1-\tau)^{1/2} \sum_{\mu} |Y_{I\mu}\left(\frac{\pi}{2}, 0\right)^2 .$$

$$\{\sigma^c_{(1)} | a_{0I,\mu}(l_1)|^2 + \sigma^c_{(2)} | a_{0I,\mu}(l_2)|^2$$

$$+ \sin\delta\,(\sigma^c_{(1)}\sigma^c_{(2)})^{1/2} [a_{0I,\mu}(l_1)\, a^*_{0I,\mu}(l_2) + a^*_{0I,\mu}(l_1)\, a_{0I,\mu}(l_2)]$$

$$- i\cos\delta\,(\sigma^c_{(1)}\sigma^c_{(2)})^{1/2} [a_{0I,\mu}(l_1)\, a^*_{0I,\mu}(l_2) - a^*_{0I,\mu}(l_1)\, a_{0I,\mu}(l_2)]\} . \qquad (2.\ 23)$$

This expression is exactly the form deduced when evaluating the inelastic integral (2. 11) by the method of stationary phase [14] [16].

At each scattering angle, the contributions of the two branchs of the deflection function (Figs. 1-2) are required; branch (1) is essentially defined by the Coulomb interaction, branch (2) by the nuclear one. The inelastic amplitudes being of opposite signs on these two branchs, the interference terms in the expressions of the elastic [16] and inelastic scattering cross-sections (2. 23) have opposite signs. So, these expressions reproduce quite well the out of phase rule observed for $\theta < \theta_r$ in the experimental results (Section 4) and its independence on the parity of the transition.

2. 3. *Analysis of the dark region of* θ_r

In the dark side of θ_r, the structure of the elastic and inelastic scattering cross-sections strongly depends on the incident energy. In experiments performed at energies about 1. 5 times the Coulomb barrier [6]-[9], the elastic and inelastic scattering drop rapidly to very low values [10]-[13]. The scattering cross-section is derived from the same formulation (2. 19) with complex conjugate values of l_1, l_2 and of $\Theta'(l_1)$, $\Theta'(l_2)$. Near θ_r, l_1 and l_2 are both nearly real and equal to the rainbow l value l_r. So, using this value and the cubic approximation of $\eta(l)$ near $l = l_r$ [10], (2. 11) reduces to the Airy approximation:

$$I^{+(r)}_{L\mu s} = e^{-iM\frac{\pi}{2}} e^{i\frac{\pi}{4}} e^{i(2\eta(l_r) - \theta l_r)} \sqrt{2}\,\pi\, l_r^{1/2}$$

$$q^{-1/3} Ai(q^{-1/3}(\theta - \theta_r))\, I^s(l_r \mu L) \tag{2. 24}$$

The rainbow scattering being the main process, the rapid fall-off of the elastic and inelastic cross-sections in the dark region is described by the behaviour of the Airy function for $\theta > \theta_r$. The inelastic cross-section for $\theta > \theta_r$ can be defined by the expression:

$$\frac{d\sigma^{I_i, I_f}}{d\Omega} = \frac{a^2(1-\tau)^{1/2}}{\sin\theta} \; \frac{2I_f+1}{2I_i+1} \, 2\pi l_r q^{-2/3} A^2 i(q^{-1/3}(\theta - \theta_r))$$

$$\sum_{\mu} \left| Y_{L\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 |a_{L\mu}(l_r)|^2 \frac{\delta(l\, I_i\, I_f)}{2L+1} \tag{2. 25}$$

At higher incident energies [15] — about 2. 7 times the Coulomb barrier —, one observes well defined oscillations in the elastic and inelastic cross-sections. In this case, the contribution of the negative branch of the deflection function (Fig. 2) is no more negligibly small in front of the rainbow amplitude. So, in (2. 10) we have:

$$I^{+}_{L\mu s} + I^{-}_{L\mu s} \cong I^{+(r)}_{L\mu s} + I^{-}_{L\mu s}(l_3) \tag{2. 26}$$

where $I^{+(r)}_{L\mu s}$ is the rainbow amplitude defined in (2. 24) and $I^{-}_{L\mu s}(l_3)$, defined with the stationary phase approximation, is

$$I^{-}_{L\mu s}(l_3) = e^{iM\frac{\pi}{2}} e^{-i\frac{\pi}{4}} \left[\frac{\pi l_3}{|\Theta'(l)|_{l_3}}\right]^{1/2} e^{i\left(2\eta(l_3) + \theta l_3 + \frac{\pi}{4}\right)}$$

$$\cdot\, I^s(l_3, \mu, L)\,. \tag{2. 27}$$

With the scattering integrals (2. 26) and the approximations (2. 24) and

(2. 27), the explicit expression of the scattering amplitude (2. 10) is:

$$\beta^{c}_{\mathrm{LM}}(\theta > \theta_r) = 4 \sum_{s} \frac{\sqrt{4\pi}}{2\mathrm{L}+1} \frac{i^{-\mathrm{L}+1}}{(\sin\theta)^{1/2}} e^{-i\mathrm{M}\left(\varphi + \frac{3\pi}{2}\right)} \cdot \mathrm{A}^{s}_{\mathrm{L}}$$

$$\sum_{\mu} \mathrm{Y}_{\mathrm{L}\mu}\left(\frac{\pi}{2}, 0\right) d^{\mathrm{L}}_{\mathrm{M},-\mu}\left(\frac{\pi}{2}\right) e^{i\mu\frac{\pi}{2}} \cdot (a_r\, e^{i\delta_r} - (-)^{\mathrm{M}} a_3\, e^{i\delta_3}) \qquad (2.\ 28)$$

where we define:

$$a_r = 2\pi\, l_r^{1/2} q^{-1/3} \mathrm{A}\, i\,(q^{-1/3}(\theta - \theta_r))\, \mathrm{I}^{s}_{\mathrm{L}\mu}(l_r)$$

$$a_3 = \left[\frac{2\pi\, l_3}{|\Theta'(l)|_{l_3}}\right]^{1/2} \mathrm{I}^{s}_{\mathrm{L}\mu}(l_3)$$

$$\delta_r = 2\,\eta(l_r) - \theta\, l_r - \frac{\pi}{4}$$

$$\delta_3 = 2\,\eta(l_3) + \theta\, l_3 - \frac{\pi}{2}. \qquad (2.\ 29)$$

In this case, the inelastic cross-section for $\theta > \theta_r$ will be given by the approximated form:

$$\frac{d\sigma^{\mathrm{I}_i \mathrm{I}_f}}{d\Omega} = \frac{a^2(1-\tau)^{1/2}}{\sin\theta} \frac{2\,\mathrm{I}_f+1}{2\,\mathrm{I}_i+1} \sum_{\mathrm{L}} \delta(\mathrm{L}\,\mathrm{I}_i\,\mathrm{I}_f) \frac{\mathrm{A}^2_{\mathrm{L}}}{2\mathrm{L}+1}$$

$$\sum_{\mu} \left| \mathrm{Y}_{\mathrm{L}\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \cdot \{2\pi\, l_r\, q^{-2/3} \mathrm{A}^2\, i\,(q^{-1/3}(\theta - \theta_r))\, |\mathrm{I}_{\mathrm{L}\mu}(l_r)|^2$$

$$+ \frac{l_3}{|\Theta'(l)|_{l_3}} |\mathrm{I}_{\mathrm{L}\mu}(l_3)|^2$$

$$- 2\,(-)^{\mathrm{L}} \cos(\delta_r - \delta_3) \left[\frac{2\pi\, l_r\, l_3}{|\Theta'(l)|_{l_3}}\right]^{1/2} q^{-1/3} \mathrm{A}\, i\,(q^{-1/3}(\theta - \theta_r))$$

$$\cdot\, \mathrm{I}_{\mathrm{L}\mu}(l_3)\, \mathrm{I}_{\mathrm{L}\mu}(l_r)\}\,. \qquad (2.\ 30)$$

For $\theta > \theta_r$, the oscillations observed in the elastic and inelastic scattering cross-sections can be described by the quantal interference effects of the rainbow scattering at each angle θ with the semi-classical scattering defined for $\Theta(l) = -\theta$. From the $(-)^L$ phase factor of the interference term in the expression (2. 30) it results that the inelastic scattering cross-sections of even and odd parity transitions present oscillations that are out of phase. If we now compare the inelastic expression (2. 20) to the elastic one:

$$\frac{d\sigma}{d\Omega}(\theta > \theta_r) = \frac{1}{k^2 \sin\theta} \{ 2\pi l_r q^{-2/3} A^2 i(q^{-1/3}(\theta - \theta_r)) \quad (2.\ 31)$$

$$+ \frac{l_3}{|\Theta'(l)|_{l_3}} - 2\cos(\delta_r - \delta_r)\left[\frac{2\pi l_3 l_3}{|\Theta'(l)|_{l_3}}\right]^{1/2} \cdot q^{-1/3} A i(q^{-1/3}(\theta - \theta_r))\}$$

we conclude that they predict oscillations which are in phase or out of phase according as the inelastic transition is even or odd. This phase rule between the elastic and inelastic cross-sections is just the inverse of the well known Blair phase rule [26] which applies in high energy diffractional like scattering. A typical example of the Blair phase rule in heavy ion is given by the scattering of ^{12}C on ^{16}O at 168 MeV [21].

3 — SCATERING POTENTIAL AND DEFLECTION FUNCTION

To describe heavy ion scattering, we use a central potential defined as the Coulomb potential superposed to a Saxon-Woods shape nuclear one

$$\nu(r) = \frac{Z_p Z_T e^2}{r} + \frac{-V_0}{1 + \exp\frac{r - R_0}{\sigma}}. \qquad (3.\ 1)$$

The deflection function is obtained by numerical evaluation of the integral:

$$\Theta(l) = \pi - 2\int_{r_m}^{\infty} \frac{\frac{(l+1/2)\hbar}{r^2}}{\left[2m(E - \nu(r) - \frac{(l+1/2)^2\hbar^2}{r^2}\right]^{1/2}} dr \qquad (3.\ 2)$$

the parameters $V_0, R_0 = r_0(A_T^{1/3} + A_p^{1/3})$ and σ of the nuclear potential being defined by a best fit of the elastic scatering cross-section. The calculations being greatly simplified by using for $\Theta(l)$ an analytic expression, we fit the deflection function obtained numerically with the parametrization form proposed by Ford and Wheeler [10]

$$\Theta(l) = \theta_r - \rho\, l^2 n \frac{l - l_a}{l_r - l_a} \tag{3.3}$$

with $\rho = q(l_r - l_a)^2$ where

$$q = \frac{1}{2}\left[\frac{d^2\Theta(l)}{d\,l^2}\right]_{l_r} \tag{3.4}$$

defines the curvature of $\Theta(l)$ at $l = l_r$.

In Fig. 1, we give the exact (3. 2) and parametrized forms of the deflection function $\Theta(l)$ used for ^{11}B on ^{208}Pb scattering at $E_{lab} = 72.2$ MeV.

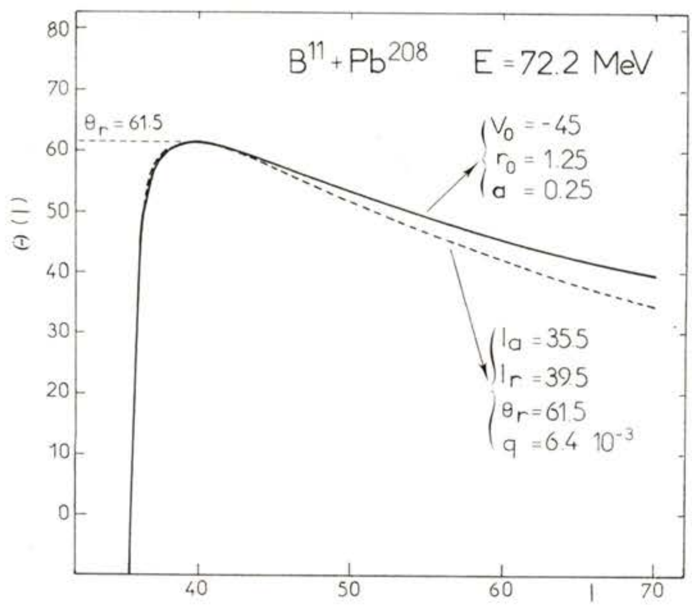

Fig. 1 — Classical deflection function used in the description of the elastic and inelastic scattering of ^{11}B on ^{208}Pb at EL = 72.2 MeV. Full curve: numerical evaluation (3.2); parametrized form (3.3).

For $l > l_r$, the decrease of the parametrized form is much too rapid with respect to the one defined by pure Coulomb field. To palliate this defect, we use in the expressions of elastic and inelastic scattering cross-sections, the Coulomb classical result $\sigma^R = \frac{\eta^{c^2}}{4 k^2 \sin^4 \theta/2}$ for $l > l_r$ i. e. on the right branch (branch 1) of the deflection function.

The deflection function used to describe the scattering of ^{12}C on ^{27}Al is defined on Fig. 2.

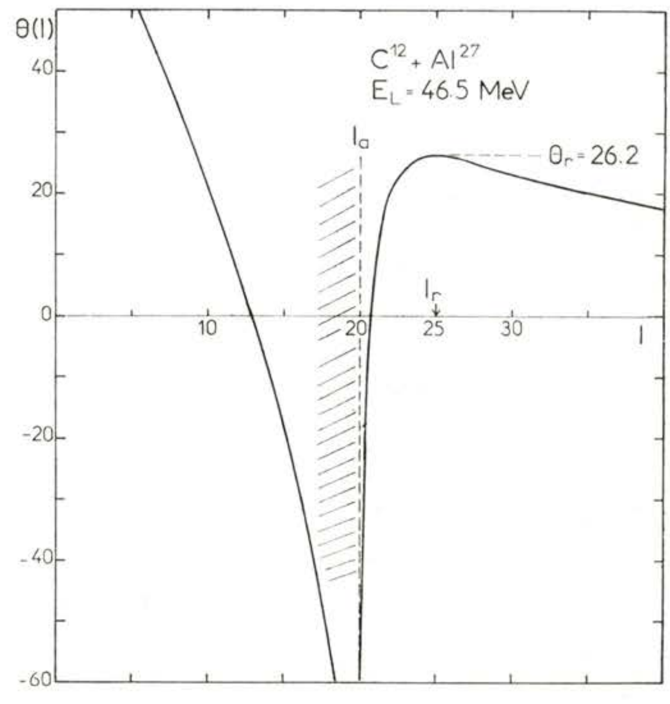

Fig. 2 — Classical deflection function used in the description of the elastic and inelastic scattering of ^{12}C on ^{27}Al at $El = 46.5$ MeV.

In such a case, the contribution from the negative branch $\Theta(l) = -\theta$ is no more negligeable against the contributions from the positive branch $\Theta(l) = +\theta$, especially in the dark region, owing to the decrease for $\theta > \theta_r$ of the contributions from $\Theta(l) = +\theta$.

4 — RESULTS

The theory outlined in Section 2 is applied to describe some experimental results in elastic and inelastic scattering of heavy ions at incident energies above the Coulomb barrier [9] — [15].

The analyzed data are:

— the elastic scattering and inelastic 3^-(2.61 MeV), 5^-(3.20 MeV), 2^+(4.10 MeV) and 4^+(4.31 MeV) excitations of ^{208}Pb in the collision with ^{11}B at laboratory incident energy of 72.2 MeV

— the elastic scattering and inelastic 2^+(4.43 MeV) transition in ^{12}C in the collision of ^{12}C on ^{27}Al at laboratory energy of 46.5 MeV.

The inelastic scattering cross-sections of ^{11}B on ^{208}Pb at 72.2 MeV laboratory energy reported on Figs. 3-4 are obtained in absolute scale with formulation (2.19); the elastic cross-sections reported are deduced from the associated expression ((2.20) in ref. [16]). The «out of phase» oscillations defined for $\theta < \theta_r$ in the elastic and inelastic scattering cross-sections are well reproduced. The observed phase rule does not depend on the parity of the inelastic transition.

The elastic and inelastic ($L = 2$) scattering cross sections of ^{12}C on ^{27}Al at 46.5 MeV laboratory energy ((2.7) times the Coulomb barrier) (Fig. 5) are obtained in absolute scale with the formulation (2.30) and (2.31) respectively. The «in phase» oscilations observed are quite well reproduced. In this case, the analysis has not been done for $\theta < \theta_r$; the behaviour of the Airy approximation used in (2.30) and (2.31) failing, badly in the lit region even when $(\theta - \theta_r)$ is only a few degrees.

Choice of parameters

The nuclear potential parameters (V_0, r_0, σ) and the parameters (θ_r, l_r, l_a, q) of the parametrized deflection function $\Theta(l)$ are obtained by best fit of the elastic cross-sections. Table I gives the values used in the present analysis. The parameter $R_c = r_c A^{1/3}$ defines the spherical distribution of charges chosen as equilibrium state for the

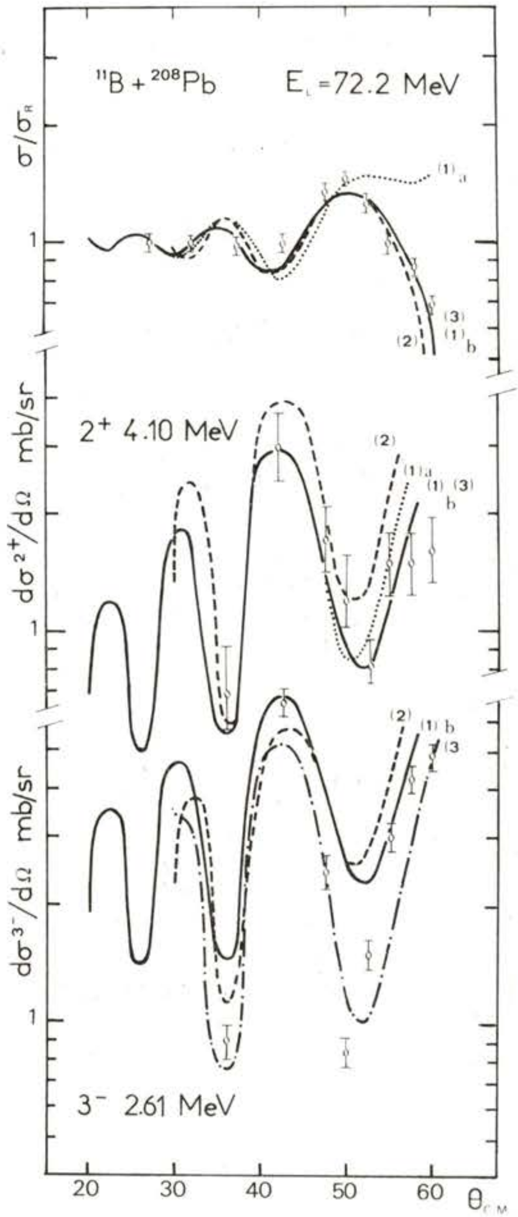

Fig. 3 — Cross-sections of the elastic and inelastic 2^+ (4.10 MeV) and 3^- (2.61 MeV) excitation of ^{208}Pb in scattering of ^{11}B on ^{208}Pb at EL = 72.2 MeV. Potentials (1) and (2) give similar fits to the data; potential (3) gives a better agreement with the experimental results [9] for the 3^- excitation. The uniform approximation (b) for the elastic and inelastic 2^+ cross-sections are compared to the asymptotic expressions (a)

target ion. (a) is the distance of closest approach of the ions on their trajectories [16]. As it is well known, a variation of the potential shape does not really affect the elastic scattering results but introduces sensitive perturbations in the inelastic one, the form factor being proportional to the derivative of the nuclear potential. The variation of the nuclear potential parameters induces variation in the relative

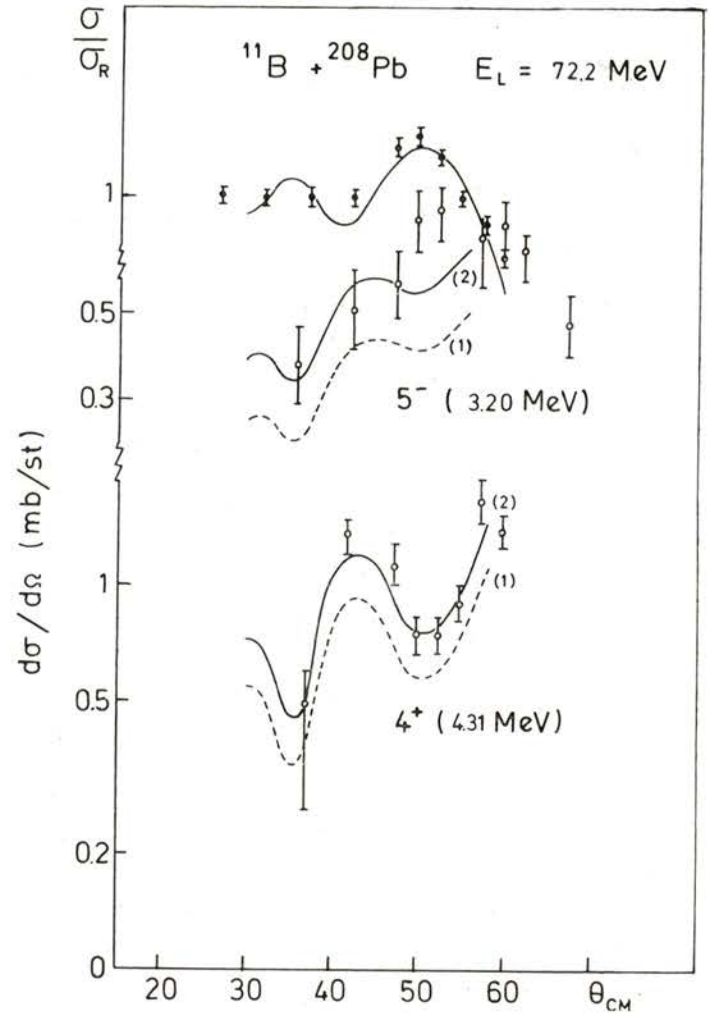

Fig. 4 — Cross-sections of elastic and inelastic 5^- (3.20 MeV) and 3^+ (4.31 MeV) excitation of ^{208}Pb in scattering of ^{11}B on ^{208}Pb at EL = 72.2 MeV. Potentials (1) and (2) are the same of Figure 3.

amplitudes of the oscillations and in the absolute scale of the inelastic results. The interference structure is very sensitive to the diffuseness of the nuclear potential and to the distance parameter (a) whose value depends on the form of the nuclear potential.

In the scattering of ^{11}B on ^{208}Pb, three potentials have been used. For set (2) the last minimum ($\theta < \theta_r$) in the inelastic cross-section is not deep enough for the 2^+ as well as for the 3^- state excitation. This choice defines too small values of the deformation parameters β_L^T (Table II) for $L = 2$ and $L = 3$.

In direct process formalism, the interference of Coulomb and nuclear excitation terms in describing the inelastic cross-section to higher multipolarity is a more drastic test to define accurate choice of the nuclear potential paramenters.

The deformation parameters β_L defined by the inelastic result only acts as a scale factor to adjust the absolute values of the inelastic cross-section. In electron and light ions inelastic scattering, the deformation of the interaction potential can be expected to be nearly the same as the charge or mass deformation of the target ion. This is not expected to be the case when the projectile size is large as it

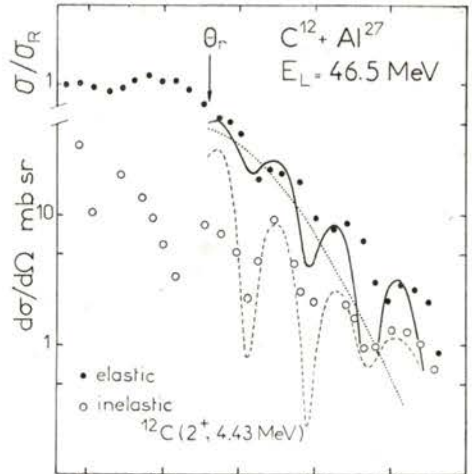

Fig. 5 — Cross-sections of elastic and inelastic 2^+ (4.43 MeV) excitation of ^{12}C in scattering of ^{12}C on ^{27}Al at EL = 46.5 MeV. The elastic cross-section is defined by expression (2.31) with (full curve) and without (dotted curve) the negative branch contribution; the inelastic one is defined by the expression (2.30) (broken curve). Experimental results are from ref. [15].

happens for heavy ion projectiles. The deformation obtained for the ion-ion potential will be smaller than the one of the target nuclear state involved. This effect has already been observed in $\alpha - \alpha$ scattering [22]. Taking into account the first order size correction [23] one may deduce the target deformation from the potential one, using the relation $\beta_L^V R_0 \equiv \beta_L^T R_T$ where $\beta_L^V, \beta_L^T, R_0, R_T$ are the deformation parameters of (L) multipolarity and the radii of the ion-ion potential and

TABEE I

	^{11}B ^{208}Pb $E_L = 72.2$ MeV									
	ion-ion potential parameters						*Deflection function parameters*			
	V_0	W_0	r_0	r_c	σ	(a)	θ_r	l_r	l_a	q
(1)	40.	0.	1.25	1.20	0.4	4.08	61.5	39.5	35.5	0.0064
(2)	50.	0.	1.20	1.20	0.6	4.	60.4	39.5	35.4	0.0052
(3)	45.	0.	1.25	1.20	0.5	4.28	61.5	39.5	35.5	0.0064
	^{12}C ^{27}AL $E_L = 46.5$ MeV									
	ion-ion potential parameters						*Deflection function parameters*			
	V_0	W_0	r_0	r_c	σ	(a)	θ_r	l_r	l_a	q
	35.	0.	1.15	1.20	0.55	1.6	26.2	25	20.5	0.0045
	$\beta_2^V = 0.30$									

of the target respectively. We use the relations $R_0 = r_0 (A_T^{1/3} + A_P^{1/3})$ and $R_T = r_T A_T^{1/3}$ (Table II). We test two descriptions of the excitation processes, they correspond to the choice of $\beta_L^V R_0$ or $\beta_L^V R_T$ as nuclear deformation parameter in the excitation potential. The potential and target deformation parameters obtained by fit of the inelastic results are in good agreement with values defined by other experimental results and theoretical DWBA analysis.

TABLE II

				Deformation parameters in ^{208}Pb				
3^- 2.61 MeV		$r^{(a)}$	r_0^T	β_L^V	β_L^T	$\beta_L^{(d)}$	$BE_{L\,e^2b^2}$	*Reference*
	(1)	1.25	$1.18^{(b)}$	0.07	—	0.102	0.49	(^{11}B, ^{11}B′) this work
	(2)	1.20	1.18	—	0.06	0.061	0.18	id.
	(3)	1.25	1.18	0.09	—	0.13	0.75	id.
other works		1.31	1.27	0.06	—	0.085	0.35	(^{16}O, ^{16}O′) [7]
		1.34	1.04	0.07	—	0.09	0.40	(^{11}B, ^{11}B′) [9]
							0.58	[24]
2^+ 4.10 MeV		$r^{(a)}$	r_0^T	β_L^V	β_L^T	$\beta_L^{(d)}$	$BE_{L\,e^2b^2}$	*Reference*
	(1)	1.25	$1.18^{(b)}$	0.0425	—	0.062	0.36	(^{11}B, ^{11}B′) this work
	(2)	1.20	1.18	—	0.048	0.049	0.23	id.
	(3)	1.25	1.18	0.06	—	0.087	0.69	id.
other works		1.31	1.27	0.03	—	0.043	0.18	(^{16}O, ^{16}O′) [7]
		1.34		0.042	—	0.06	0.35	(^{11}B, ^{11}B′) [9]
							0.30	[24]
5^- 3.20 MeV		$r^{(a)}$	r_0^T	β_L^V	β_L^T	$\beta_L^{(d)}$		*Reference*
	(1)	1.25	$1.18^{(b)}$	0.036	—	0.05		(^{11}B, ^{11}B′) this work
	(3)	1.20	1.18	—	0.06	0.06		id.
other works		1.34	1.04	—	0.05	0.06		(^{11}B, ^{11}B′) [9]
		—	—	—	0.043	0.055		(p, p') [24]
4^+ 4 31 MeV		$r^{(a)}$	r_0^T	β_L^V	β_L^T	$\beta_L^{(d)}$		*Reference*
	(1)	1.25	$1.18^{(b)}$	0.05	—	0.07		(^{11}B, ^{11}B′) this work
	(3)	1.20	1.18	—	0.08	0.08		id.
other works		1.34	1.04	—	0.07	0.09		(^{11}B, ^{11}B′) [9]
		—	—	—	0.062	0.08		(p, p') [24]

(*a*) $r = r_0$ in case (1) and (3); $r = r^T$ in case (2) with $\beta_L^V r_0 (A_P^{1/3} + A_T^{1/3}) = \beta_L^T r^T A^{1/3}$.

(*b*) r_0^T is the value difined in [25]:

(*d*) β_L deformation parameter difined by $\beta_L^V R_0 = \beta_L r_0 A_T^{1/3}$ or by $\beta_L^T R^T = \beta_L r_0^T A_T^{1/3}$.

5 — CONCLUSION

The aim of this paper has been to gain insight into the physical understanding of the different oscillatory behaviours which are observed in the elastic and inelastic heavy ions scattering cross-sections.

In the lit region, the observed phase rule results from the quantal interference effect between the classical trajectories deviated by one side of the nucleus.

In the dark region, a different oscillatory behaviour appears; it is due to an additional interference effect resulting from the trajectories deviated by the opposite side of the nucleus. At higher energies, when Coulomb effects are negligeable, the scattering is dominated by Fraunhøfer diffraction and then, the Blair phase rule applies.

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