PAULI CORRECTION TO THE DEUTERON FOLDED POTENTIAL (*)

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ABSTRACT — The correction to the deuteron-nucleus effective interaction in deuteron elastic scattering resulting from the Pauli exclusion principle was calculated following the method proposed by Soper [5], using the infinite nuclear matter approximation and the separable nuclear potential of Yamaguchi [14]. The resulting Pauli potential is repulsive and has a magnitude of few percent of the deuteron folded potential real part. Its radial dependence, determined with the local density approximation, shows a pronounced peak at the nuclear surface.

RESUMÉ — La correction resultante du principe de l'exclusion de Pauli qui s'applique à l'interaction effective deuteron-noyau pour la diffusion elastique fut calculée par la methode de Soper [5], utilisant l'approximation de matière nuclaire infinie et le potentiel nucleaire separable de Yamaguchi [14]. Le potentiel de Pauli ainsi obtenu est repulsif avec une intensité de l'ordre de quelques pour-cent de la partie réelle du potentiel du deuteron de Watanable [6]. La dependence radiale de ce potentiel, determinée avec l'approximation de densité locale, nous montre un pic pronouncé à la surface nucléaire.

1 — INTRODUCTION

The Pauli exclusion principle plays a dominant role in atomic collisions, due to the exchange forces set up in the overlapping electron clouds of the colliding atoms. This effect has been recognized for several years, and recently reported by Laubert and Brandt [1]. An analogous but less pronounced effect will be present in nuclear reactions involving multinucleon bound projectiles and targets. However even for the nucleon-nucleus optical potential the calculation of Pauli

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exclusion principle effects presents considerable difficulties. This problem has been considered by Bell and Squires [2] and by Van Giai, Sawicki and Vinh Mau [3] in terms of a linked cluster expansion neglecting the target recoil energy and assuming the ground-state to be non-degenerate. The same method has been used by Junkin and Villars [4] to derive an expression for the deuteron optical potential. The linked cluster perturbation series for the effective interaction 2 nucleons-nucleus has two terms: the first is a one-body operator, where the leading term is just the Hartree-Fock (H-F) potential, and corresponds to the sum of two single-nucleon optical potentials calculated off the energy shell at roughly half the deuteron kinetic energy; the second is a two-body operator, part of which comes from the exclusion principle. Soper [5] derived from the full antisymmetrized hamiltonian an equation which describes two nucleons outside a closed shell interacting with each other via the 2-nucleon interaction and with the core through their H-F potentials. The solution of this equation is a 2-nucleon state containing only particle-states ortogonal to the bound H-F orbitals. Note however that the most general 2-nucleon state satisfying the asymptotic boundary condition of an incoming deuteron with momentum $\vec{K}$ also has hole-states in its expansion. The effective interaction in that equation is the sum of the H-F potentials plus the 2-body potential which is the lowest order approximation to the 2-body potential of Junkin and Villars [4]. Neglecting the coupling to the break-up channel, the correction to the folded potential of Watanable [6] due to the Pauli exclusion principle is just the mean value of a 2-body potential in the deuteron internal wave function.

Using this approximation Johnson and Soper [7] have calculated deuteron break-up effects in deuteron stripping reactions using an adiabatic model and obtained improved agreement with differential cross section data. Recently, Gambhir and Griffin [8, 9] investigated explicitly the deuteron break-up due to Pauli exclusion effects in deuteron elastic scattering and Austern [10] proposed a simple procedure for the calculation of antisymmetrization effects. Using this model Gambhir and Griffin [11] have discussed the qualitative features of Pauli break-up from the configuration space viewpoint.

In the present paper we analyse the Pauli correction to the deuteron folded potential, proposed by Soper [5], in particular the variation of its radial shape with deuteron incident energy and target nucleus mass number.

2 — PAULI CORRECTION TO THE DEUTERON FOLDED POTENTIAL

2. 1. *General formalism*

The terms in the deuteron-nucleus effective interaction corresponding to Pauli exclusion principle effects [5] can be written as

$$V_p(1,2) = [E - h(1) - h(2)]\,[1 - P(1)\,P(2)] + \qquad (1)$$
$$[P(1)\,P(2)\,v(1,2)\,P(1)\,P(2) - v(1,2)]$$

where E is the total energy of the system and $v(1,2)$ the 2-nucleon interaction. $P(i)$ is the projection operator into the eigenstates of the 1-body hamiltonian.

$$h(j) = T + V_H\,,\; j = 1,2$$

belonging to the eigenvalue $\varepsilon_i > \varepsilon_F$. Here V_H is the self-consistent H-F potential for A nucleons and ε_F the Fermi energy. In order to obtain the Pauli correction to the deuteron-nucleus folded potential we must calculate the average of the interaction $V_p(1,2)$ over the internal motion of the deuteron. The equivalent local potential will be calculated considering the momentum space representative of the $V_p(1,2)$ interaction

$$\langle \vec{K}\,\varphi_0 \,|\, V_p(1,2) \,|\, \vec{K}'\,\varphi_0 \rangle$$

where φ_0 is the deuteron internal wave function. We now represent the nucleus by infinite nuclear matter which, strictly speaking, means that the correction only applies to the nucleus center. In this approximation the H-F states became plane waves and the projection operators into particle states have the form

$$P(j) = \sum_{\sigma_i\,\tau_i} \int_{k_i > k_F} d\vec{k}_i \,|\, \vec{k}_i\,\sigma_i\,\tau_i \rangle \langle \vec{k}_i\,\sigma_i\,\tau_i \,| \qquad (3)$$

where σ_i and τ_i represent the spin and isospin variables. Further-

more the H-F potentials must be diagonal in momentum space. We choose as usual [12] a quadratic dependence in $\vec{k}_i$

$$<\vec{k}_1 \,|\, \mathrm{V}(1) \,|\, \vec{k}'_1> = \delta(\vec{k}_1 - k'_1)\,(\omega_0 + \omega_1 k_1^2) \tag{4}$$

which is equivalent to assume an effective nucleon mass m^* inside nuclear matter given by

$$\frac{1}{2\,m^*} = \frac{1}{2\,m} + \omega_1 \tag{5}$$

where m is the nucleon mass. The constants ω_0 and ω_1 are estimated from the energy dependence of the depth of the real part of the phenomenological nucleon optical potential $\mathrm{V}(\mathrm{E_N})$. Using the relation

$$\mathrm{E_N} = \frac{\hbar\, k_1^2}{2\,m} + \mathrm{V}(\mathrm{E_N})$$

where $\mathrm{E_N}$ is the kinetic energy of the nucleon outside the nucleus and the average nucleon optical potentials of Bechetti and Greenlees [13] we obtain

$$\omega_0 = -(81.10 + 0.29\ \mathrm{ZA}^{-1/2})\ \mathrm{MeV}$$
$$\omega_1 = 9.75\ \mathrm{MeV}\ fm^2$$

where A and Z are, respectively the mass and atomic number of the target.

A separable potential to represent the 2-nucleon interaction is particularly convenient in view of the structure of eq. (1). We use the potential of Yamaguchi [14] which fits the low energy 2-nucleon data. This potential also has the advantage that it gives a Hulthén type wave function for the deuteron which is particularly convenient for the required momentum space integration. The restricted domain of integration for the individual nucleon momenta k_i imposes conditions on the deuteron internal momentum k which is therefore subjected to the relations;

$$\frac{\mathrm{K}}{2} - k_\mathrm{F} \geqq k \geqq \frac{\mathrm{K}}{2} + k_\mathrm{F}$$

and

$$\frac{\mathrm{K}}{2} - k_\mathrm{F} < k < \frac{\mathrm{K}}{2} + k_\mathrm{F}$$

where in the latter case the angle θ between K and k must satisfy

$$\frac{K^2 - 4k_F^2}{4Kk} - \frac{k}{K} < \cos\theta < \frac{K^2 - 4k_F^2}{4Kk} + \frac{k}{K}.$$

We represent by an over bar the integrations in k over the allowed momentum space and by $\overline{N}$ the integral of the Hulthén wave function over that space. Using this notation we have;

$$<\vec{K}\,\varphi_0 \,|\, V_p(1,2) \,|\, \vec{K}'\,\varphi_0> = (\vec{K} - \vec{K}')\, V_p(K)$$

where

$$V_p(K) = \left[E_0 - \frac{\hbar^2 K^2}{4m} - 2\left(\omega_0 + \omega_1\left(\frac{K^2}{4} + <k^2>\right)\right)\right](1-\overline{N})$$

$$+\left[\overline{<v>} + \left(\frac{\hbar^2}{m} + 2\omega_1\right)\overline{<k^2>} - \left(<v> + \frac{\hbar^2}{m}<k^2>\right)\overline{N}\right]$$

$$-2\omega_1\overline{N}<k^2> \qquad (8)$$

and E_0 is the deuteron incident energy which is the difference between the total energy and the deuteron binding energy ε_0. The latter can be written

$$\varepsilon_0 = <v> + \frac{\hbar^2}{m}<k^2>$$

where

$$<v> = \int d\vec{k}\, d\vec{k}'\, \varphi_0(k)\, v(k,k')\, \varphi_0(k')$$

$$<k^2> = \int d\vec{k}\, \varphi_0(k)\, k^2\, \varphi_0(k).$$

The choice of K corresponding to a given deuteron incident energy E_0 is made in a self consistent way [15] through the equation

$$E_0 - \frac{\hbar^2 K^2}{4m} - 2\left[\omega_0 + \omega_1\left(\frac{K^2}{4} + <k^2>\right)\right] - V_p(K) = 0. \qquad (10)$$

Using eqs. (8), (9) and (10) we obtain

$$V_p(E_0) = \varepsilon_0^* - \varepsilon_0' \qquad (11)$$

where

$$\varepsilon_0^* = \frac{\overline{< v >}}{\overline{N}} + \left(\frac{\hbar^2}{m} + 2\,\omega_1 \right) \frac{\overline{< k^2 >}}{\overline{N}} \tag{12}$$

may be interpreted as an internal energy of the neutron proton system inside nuclear matter and

$$\varepsilon_0' = < v > + \left(\frac{\hbar^2}{m} + 2\,\omega_1 \right) < k^2 > \tag{13}$$

is the binding energy of a «deuteron» with an effective nucleon mass given by eq. (5). By substitution of eq. (11) into eq. (10) we finally get

$$E = \left(\frac{\hbar^2}{m} + 2\,\omega_1 \right) \frac{K^4}{4} + 2\,\omega_0 + \varepsilon_0^* . \tag{14}$$

Thus the total energy of the system «deuteron inside nuclear matter» is the sum of the deuteron center of mass energy corresponding to a nucleon effective mass m^*, the potential energy and the internal energy ε_0^*. We emphasize that the Pauli potential V_p is a function of the incident energy E_0 through ε_0^*. In fact note that the allowed momentum space for the k integrations is a function of K and therefore of E_0.

The eq. (11) has the same structure of the Pauli potential of Gambhir and Griffin [8]

$$V_x = Q\,H_i\,Q - H_i \tag{15}$$

where Q projects into the allowed momentum space and H_i is the free deuteron hamiltonian.

We can investigate the radial dependence of the Pauli potential V_p using the local density approximation. The Fermi momentum k_F is then determined by the value of the nuclear density form factor at radius R through the relation

$$k_F^3 = \frac{3\,\pi^2}{2}\,\rho\,(R) . \tag{16}$$

For consistency we also assume that ω_0 and ω_1 in eq. (4) have the same radial dependence as the nucleon optical potential of Bechetti and Greenless [13].

2. 2. *Results of calculation*

The present calculations of the Pauli potential V_p given by eq. (11) were made using for the deuteron a Hulthén wave function with parameter $\beta = 1.36\,\mathrm{fm}^{-1}$, the nucleon optical potentials from Ref. [13] and the nuclear density form factor $\rho(R)$ from Ref. [16] to determine its radial dependence. As shown in Fig. 1 V_p is repulsive and decreases

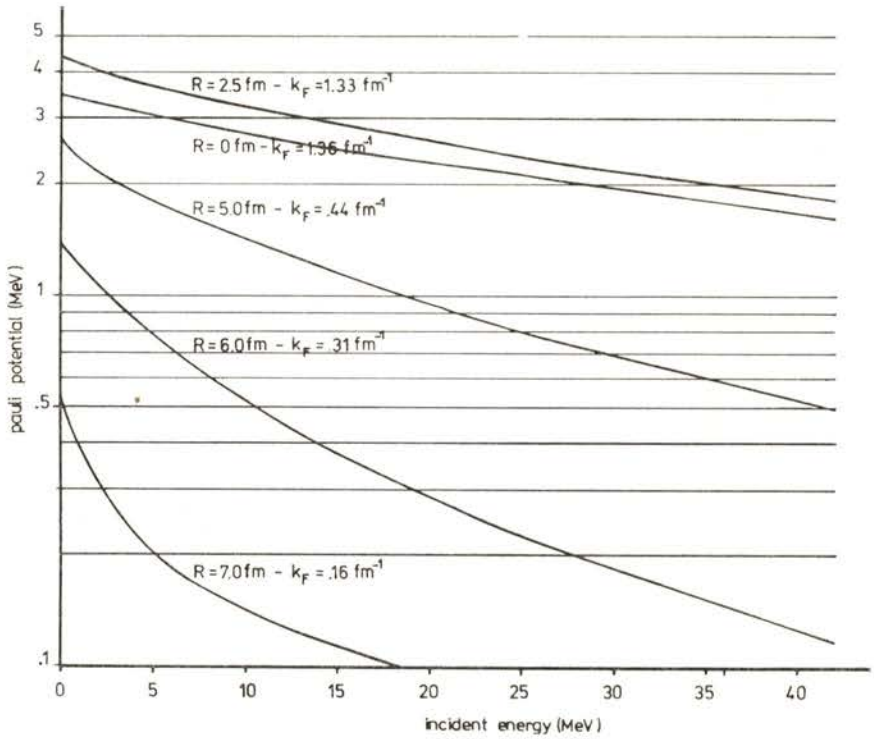

Fig. 1 — Variation of the Pauli potential in ^{40}Ca with deuteron incident energy for different values of k_F. The values of R are determined from eq. (16).

roughly exponentially with incident deuteron energy. We find that V_p has generally a larger radius than the nuclear optical potential. The difference is about 0.7 fm in ^{40}Ca for $E_0 = 10\,\mathrm{MeV}$ and increases for smaller energies. A distinctive feature of V_p is a depression in the nuclear center.

The various contributions to the Pauli potential for ^{40}Ca at $E_0 = 5$ MeV are plotted in Figs. 2 and 3 for ω_1 equal to 0,5, 9.75 and 13.2 MeV fm^2. The quantities ε_0^* and ε_0' have different radial forms. ε_0' being proportional to the nucleon optical potential since it depends linearly on ω_1. The two terms in ε_0^* have a ratter complicated dependence on R and ω_1 but their sum always has a mean radius

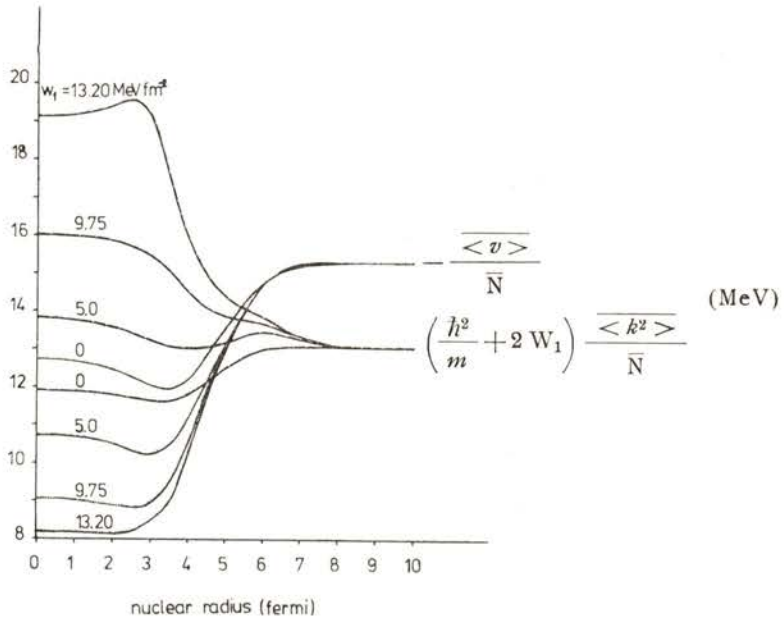

Fig. 2 — Radial dependence of the two terms in the expression (12) which gives the internal energy of the neutron-proton system inside nuclear matter for different vallues of ω_1. The target is ^{40}Ca and the deuteron incident energy $E_0 = 5$ MeV.

greather then ε_0' as shown in Fig. 3. Further, with $\omega_1 = 0$ and 13.2 MeV fm^2, ε_0^* presents a little surface peak, which is not present at intermediate values of ω_1. These two facts, namely the difference in radius and the surface peak in ε_0^*, lead to a pronounced surface peak in the Pauli potential. The increase of V_p with ω_1 shown in Fig. 4 is mainly due to the pronounced dependence of the ε_0^* term.

The behaviour of V_p as a function of the deuteron incident energy is represented in Fig. 5. As expected, V_p decreases with deuteron

incident energy, but this variation is less pronounced in the nuclear interior. In fact, in this region of space K varies slowly with E_0, because it is mainly determined by $2\omega_0$ (eq. (10)). However, at the nuclear surface this variation is much more pronounced, and therefore the domain of integration in k is drastically increased.

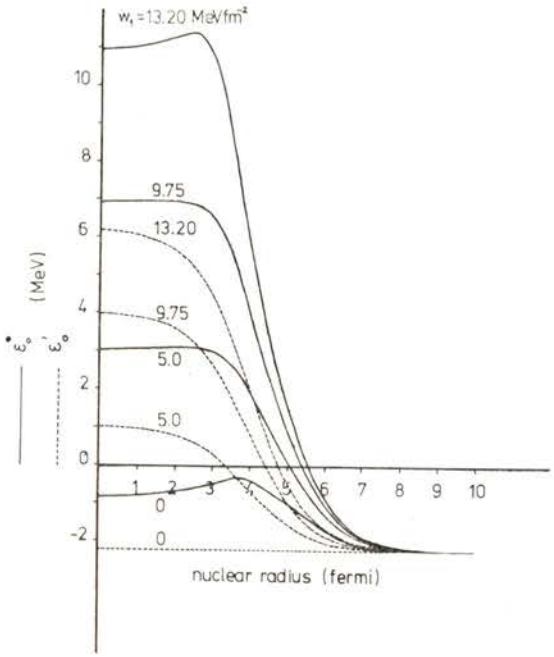

Fig. 3 — Radial dependence of the two terms in the expression (11) which gives the Pauli potential V_P for different values of ω_1. The target and incident energy is as in Fig. 2.

Fig. 6 represents the radial dependence of V_P for various nuclei. We note that the variation with A is less pronounced in the nuclear interior. This is due to the fact that V_P is invariant with A in the infinite nuclear matter approximation if we neglect the weak dependence of ω_0 on A and Z.

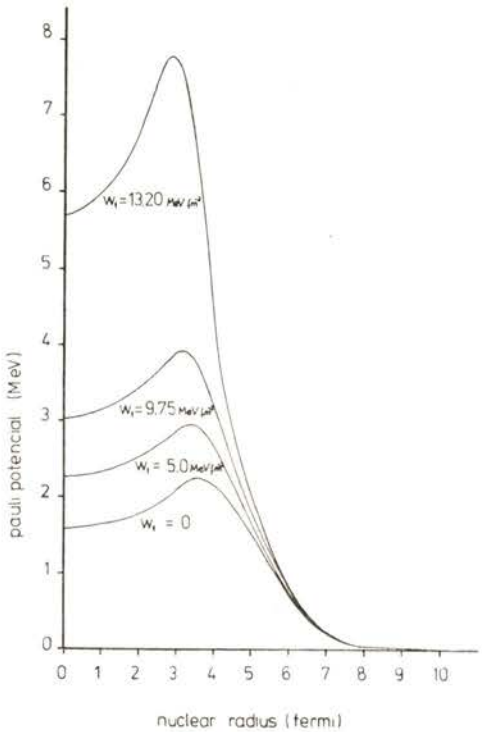

Fig. 4 — Radial dependence of the Pauli potential in ^{40}Ca for a deuteron incident energy $E_0 = 5$ MeV and for different values of ω_1.

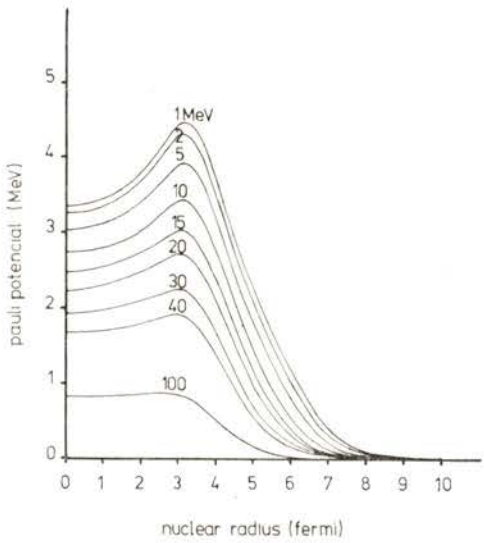

Fig. 5 — Radial dependence of the Pauli potential in ^{40}Ca for different deuteron incident energies and assuming $\omega_1 = 9.75$ MeV fm^{-2}.

3 — CONCLUSIONS

The Pauli correction to the deuteron folded potential is found to be repulsive and to decrease with deuteron incident energy as it should be expected from simple physical grounds. The same type of behaviour is reported by Gambhir and Griffin [11]. The magnitude of V_p is of the

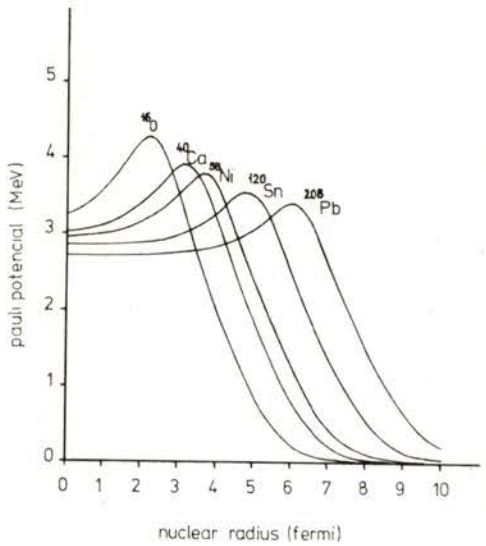

Fig. 6 — The Pauli potential in different target nuclei for a deuteron incident energy $E_0 = 5$ MeV and assuming $\omega_1 = 9.75$ MeV fm^{-2}.

order of 5% of the depth of the real part of the folded potential [17] and its radius is larger. As regards the radial dependence we find a marked surface peak particularly in light nuclei. This therefore does not give support to the assumption of Ref. [11] of uniformity of V_p over the nuclear volume.

The deuteron becomes unbound at a radius where the nuclear density is only few percent of the value in the center. This radius is found to be appreciably sensitive to the coefficient ω_1 which gives the nucleon optical potential energy dependence. For the value of ω_1 taken from [13] the deuteron reaches an unbound state even for relatively

high energies of the order of 100 MeV. However if we take $\omega_1 = 0$ the deuteron is never unbound for an incident energy $E_0 > 5$ MeV.

We note that in the calculations of Ref. [9] the deuteron is unbound in nuclear matter unless it has a kinetic energy of the order of 100 MeV. These predictions are significantly altered in the present more realistic calculations of the intrinsic energy of the $n-p$ system inside nuclear matter.

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REFERENCES

[1] R. LAUBERT and W. BRANDT, *Phys. Rev. Letters* **24**, 1037 (1970).
[2] J. S. BELL and E. J. SQUIRES, *Phys. Rev. Letters* **3**, 96 (1959).
[3] N. VAN GIAI, J. SAWICKI and N. VINH MAU, *Phys. Rev.* **141**, 913 (1966).
[4] W. F. JUNKIN and F. VILLARS, *Ann. Phys.* (N. Y.) **45**, 93 (1967).
[5] P. J. R. SOPER, Ph. D. Thesis, University of Surrey, unpublished.
[6] S. WATANABLE, *Nucl. Phys.* **8**, 484 (1958).
[7] R. C. JOHNSON and P. J. R. SOPER, *Phys. Rev.* **C1**, 976 (1970).
[8] B. L. GAMBHIR and J. J. GRIFFIN, *Phys. Rev.* **C5**, 1856 (1972).
[9] B. L. GAMBHIR and J. J. GRIFFIN, *Phys. Rev.* **C7**, 590 (1973).
[10] N. AUSTERN, *Phys. Lett.* **46B**, 49 (1973).
[11] B. L. GAMBHIR and J. J. GRIFFIN, *Phys. Lett.* **50B**, 407 (1974).
[12] K. KIKUCHI and M. KAWAI, *Nuclear Matter and Nuclear Reactions* (Amsterdam: North-Holland, 1968), pag. 5.
[13] F. D. BECHETTI and G. W. GREENLESS, *Phys. Rev.* **182**, 1190 (1969).
[14] Y. YAMAGUCHI, *Phys. Rev.* **95**, 1628 (1954).
[15] D. J. THOULESS, *Nucl. Phys.* **75**, 128 (1966).
[16] P. E. HODGSON, *Nuclear Reaction and Nuclear Structure* (London: Oxford University Press, 1971), pag. 28.
[17] J. R. ROOK, *Nucl. Phys.* **61**, 219 (1965).