GENERATOR COORDINATE METHOD AND QUANTUM FLUID DYNAMICS (*)

J. DA PROVIDÊNCIA
Departamento de Física, Universidade de Coimbra

We wish to discuss the equivalence between the Generator Coordinate Method [1] (GCM) and Quantum Fluid Dinamics (QFD).

We considerer the Hamiltonian

$$H = \sum_{i=1}^{n} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i \neq j} v(\vec{r}_i - \vec{r}_j)$$

and the Hartree-Fock state

$$| \Phi_o > = \prod_{\vec{p}\,(p \leq p_F)} c^+_{\vec{p}} \, | 0 >$$

where $c^+_{\vec{p}}$ is the creation operator for a fermion with momentum $\vec{p}$ and $| 0 >$ is the absolute vacuum. Let

$$b^+_{\vec{k}} = \sum_{\vec{p} \in D_{\vec{k}}} c^+_{\vec{p}+\vec{k}} \, c_{\vec{p}}$$

denote the generator for density deformations [2]. Here $D_{\vec{k}}$ is the domain defined by $p \leqslant p_F$, $| \vec{p} + \vec{k} | > p_F$. The time evolution of the deformed Slater determinant

$$| \Phi(z) > = \exp \Big[\sum_{\vec{k}} z_{\vec{k}} \, b^+_{\vec{k}} \Big] \, | \Phi_o >$$

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is determined by the variational principle

$$\delta [< \Phi | H | \Phi > / < \Phi | \Phi >] - i [< \delta \Phi | \dot{\Phi} > - < \dot{\Phi} | \delta \Phi >] = 0$$

which, to leading order in the small quantities z_k, z_k^*, may be written

$$\sum \{ i \, (\delta \, z^*_{\vec{k}} \, \dot{z}_{\vec{k}} - \dot{z}^*_{\vec{k}} \, \delta \, z_{\vec{k}}) \, N_k$$

$$- \delta [A_k \, z^*_{\vec{k}} \, z_{\vec{k}} + \frac{1}{2} B_k \, (z_{\vec{k}} \, z_{-\vec{k}} + z^*_{\vec{k}} \, z^*_{-\vec{k}})] \} = 0 \, ,$$

where

$$A_k = < \Phi_o | \, b_{\vec{k}} \, (H - E_o) \, b^+_{\vec{k}} \, | \, \Phi_o > ,$$

$$B_k = < \Phi_o | \, b_{\vec{k}} \, b_{-\vec{k}} \, H \, | \, \Phi_o > ,$$

$$E_o = < \Phi_o | \, H \, | \, \Phi_o > ,$$

$$N_k = < \Phi_o | \, b_{\vec{k}} \, b^+_{\vec{k}} \, | \, \Phi_o > .$$

We introduce now the quantities

$$\rho = \sum_{\vec{k}} N_k \, (z_{\vec{k}} + z^*_{-\vec{k}}) \, e^{i \vec{k} \cdot \vec{r}} / V$$

$$\phi = \sum_{\vec{k}} (z_{\vec{k}} - z^*_{-\vec{k}}) \, e^{i \vec{k} \cdot \vec{r}} / (2 \, m \, i) .$$

The variational principle becomes

$$m \int (\delta \, \rho \, \dot{\phi} - \delta \, \phi \, \dot{\rho}) \, d^3 \vec{r} + \frac{m \, n}{2 \, V} \, \delta \int \nabla \phi \cdot \nabla \phi \, d^3 \vec{r}$$

$$+ \delta \iint \rho \, (\vec{r}) \, \rho \, (\vec{r}') \, u \, (\vec{r} - \vec{r}') \, d^3 \vec{r} \, d^3 \vec{r}' = 0$$

where $$u \, (\vec{r}) = \sum_{\vec{k}} (A_k - B_k) \, e^{i \vec{k} \cdot \vec{r}} / N_k^2$$

and n denotes the particle number.

This may be recognized as the variational principle of QFD [3], for small deviation from equilibrium. The conditions for validity of QFD and of GCM in the version described here [2], are, therefore, essentially the same.

REFERENCES

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