RELATIVISTIC EQUATIONS AND THE STRUCTURE OF MESONS (*)

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ABSTRACT — We present some preliminary results concerning the application of a relativistic model to the psi-family. This model differs from a previous one, based on the Bethe-Salpeter equation in instantaneous ladder approximation, in that it includes an anomalous magnetic moment at the quark-gluon vertex. With this improvement we get good numerical results for the hyperfine splitting between the Ψ (Ψ') and the η_c (η'_c).

Numerical estimates are also made for the light mesons. The aim is to fit the whole meson family with a single type of potential. We are still far from solving this problem, but we put forward some arguments showing that our expectations seem to be reasonable.

1 — INTRODUCTION

The charmed quark-antiquark ($c\bar{c}$) interpretation of Ψ (3.1) and Ψ' (3.7) has received considerable support after the observation of monoenergetic γ-ray transitions from Ψ and Ψ' to states easily interpreted as P-wave states of the $c\bar{c}$ system and as ${}^1S_0$ partners of the ${}^3S_1$ states (the Ψ and Ψ').

While some quark models have been able to accomodate the details of the P-wave spectrum [1], the large ground-state hyperfine splitting $E(\Psi)-E(\eta_c) \sim 270$ MeV still remains unexplained. To solve this problem, Schnitzer [2] proposed an effective quark-gluon anomalous moment, within the context of the linear potential model. Unfortunately, although in this model the $\Psi-\eta_c$ splitting comes with the right order of magnitude, the ratio $R=\{E(2^{++})-E(1^{++})\}$ $\{E(1^{++})-$

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$-E(0^{++})\}\simeq 0.89$, too large when compared with the experimental value $R\simeq 0.40$.

In this paper we show how, in the context of a certain relativistic model, previously developed by Kellet, Moorhouse and the author [1], a quark-gluon anomalous magnetic moment may help us in solving the hyperfine splitting puzzle without destroying the very good results obtained, in that model, for the P-wave states.

In the remaining of this section we describe the main ingredients of our model and quickly summarize some of the results obtained for charmonium. Then, in the next section, we introduce the quark-gluon anomalous moment into the model and perform some rough numerical estimates, both for charmonium and for the light-mesons. Our ultimate goal is to fit the whole meson family with one single type of potential. Having found that such a goal seems reasonable, in section 3 we solve our relativistic equations, using the accurate numerical methods described in ref. [1].

Our general framework is provided by the Bethe-Salpeter equation, in ladder approximation with a static potential. The equation takes the form

$$(E-H_a+H_b)\,\phi(\vec{q})=(\Lambda_+^a\,\Lambda_-^b-\Lambda_-^a\,\Lambda_+^b)\,\beta^a\,\beta^b\int V(\vec{q}-\vec{q}\,')\,\phi(\vec{q}\,')\,d^3\vec{q}\,' \qquad (1)$$

where $\quad H_a=\vec{\alpha}_a\cdot\vec{q}_a+\beta_a m_a,\quad \Lambda_{\pm}^a=(W_a\pm H_a)/2\,W_a$

and $\quad W_a=\sqrt{\vec{q}_a^{\,2}+m_a^2}\quad (\vec{q}_a=-\vec{q}_b\equiv\vec{q}).$

In the following applications, and for simplicity, we shall supress the pair creation term, $\Lambda_-^a\,\Lambda_+^b$. [1] From the remaining equation the Fermi-Breit hamiltonian can be recovered, using the Foldy-Wouthuysen transformation.

The second ingredient of our model is an expansion of the amplitudes $\phi(\vec{q})$ in terms of the sixteen Dirac matrices. It is possible to do this and to select, among the terms of such an expansion, those appropriate to the state being studied [3].

Finally, we have to choose our potential. Following the general practice, we use a superposition of a 4-vector Coulomb potential (as suggested by asymptotic freedom, in the one-gluon exchange approximation), with a linear 4 – scalar potential, responsible for the confinement of quarks. The precise form of our potential is given by

$$V(r) = [\lambda(r-a) - \gamma^{\mu a}\gamma_{\mu_b}(3/4\ \alpha_s)\ 1/r]\exp(-\mu r) + C \qquad (2)$$

with a, μ and C constants. The factor exp (-μr) acts as the screening effect described by Kogut and Susskind [4]. Without it we would need special limiting procedures to solve the equations for energy levels above 2 m (m = quark mass) as discrete bound states.

The strong coupling constant α_s is assumed to vary with the relative momentum of the two quarks as again suggested by asymptotic freedom. The formula for α_s has not been deduced for the case of two momenta, $\vec{q}$ and $\vec{q}'$, and our application is only approximate. We took

$$\alpha_s'(\sqrt{q'q}) = \alpha_s(\bar{q})\left(1 - (50/12\pi)\,\alpha_s\,L_n(\bar{q}/\sqrt{q\,q'})\right)^{-1}, \qquad (3)$$

where $\bar{q} \sim 1$ GeV.

With the model just described, we were able to find the correct positions for the P-Wave states, as well as good values for the $\Psi' \to {}^3P_J + \gamma$ decays. We also predicted the Ψ''_D at its correct position, 3.77 GeV. However, we were not able to get the correct positions for the pseudoscalars, η_c and η_c'.

Applying our equations to the light-mesons (quark masses of the order of 300 MeV), and putting δ, A_1 and A_2 at their approximate positions, we found that the ρ-π splitting was 3 times smaller than the experimental value. Although we are dealing here with light quarks, the situation at first sight appears to be much the same as with charmonium, suggesting that the same improvement in the model might simultaneously solve the hyperfine splitting problem in both families of mesons. The improvement is described in the next section.

2 — THE QUARK – GLUON ANOMALOUS MOMENT

It has been shown by Schnitzer [2] that the consideration of an effective quark-gluon anomalous vertex would lead to a qualitative explanation of the large Ψ-η_c hyperfine splitting. It is assumed that in the ladder approximation of the Bethe-Salpeter equation, with a static potential, to lowest order in v/c the effective quark-gluon vertex is given by

$$\Gamma^{\mu} = \gamma^{\mu} - (k / 2m)\, \sigma^{\mu\nu} q_{\nu}\,, \tag{4}$$

where k is the effective quark-gluon anomalous magnetic moment, and $\sigma^{\mu\nu} = 1/2\,[\gamma^{\mu}\ \gamma^{\nu}]$. It was further assumed that k was dependent on the quark-antiquark separation, being of O (α_s) at short distances and of O (1) at large distances.

Without questioning this argument, we just want to show that a simple phenomenological alternative is possible, within our model, if we assume that k comes from the short-distance part of the potential; that is, we take

$$V_{\text{Coulomb}} \sim \Gamma^{\mu a}\ 1/r\ \Gamma_{\mu_b}\,, \tag{5}$$

Γ^{μ} given by (4).

The introduction of Γ^{μ} makes a lot of difference in our equations, and it is reflected in the form of the Fermi-Breit terms, in the following way:

(a) Spin Orbit term : $\dfrac{(3+4k)}{2m^2}\dfrac{1}{r}\dfrac{dV}{dr}\,\mathrm{L.S}$

(b) Spin-Spin term : $\dfrac{(1+k)^2}{2m^2}\,\sigma_1 . \sigma_2\ \nabla^2\ \mathrm{V}\ (r)$

(c) Tensor term : $\dfrac{(1+k)^2}{2m^2}\left(\dfrac{d^2 V}{dr^2} - \dfrac{1}{r}\dfrac{dV}{dr}\right)\mathrm{S}_{12}$

Taking $k \sim 1$ the strenght of (b) and (c) increases faster than (a), and this is the important point for our purpose (remember the P-wave splittings are mainly controled by the spin-orbit term, while (b) is dominant for the Φ-η_c and ρ-π hyperfine splittings).

The introduction of k will not affect the possibility of getting the right value for R. We can see this with the help of the approximate treatment introduced in ref. [2].

Define the following matrix elements for the P-wave states, where by H_s we mean the spin dependent part of the Fermi-Breit hamiltonian, and where for the 4-scalar linear part of the potential ($1 \otimes 1$) only the L.S term is included (with $k=0$):

$\Gamma^{\mu} \otimes \Gamma_{\mu}$

$$<J=2 \mid H_s \mid J=2> = \alpha_s <r^{-3}> m\left[(3+4k)/2-(1+k)^2/10\right]$$

$$<J=1 \mid H_s \mid J=1> = \alpha_s <r^{-3}> m\left[-(3+4k)/2+(1+k)^2/2\right]$$

$$<J=0 \mid H_s \mid J=0> = \alpha_s <r^{-3}> m\left[-(3+4k)-(1+k)^2\right] \quad (6\,a)$$

$1 \otimes 1$

$$<J=2 \mid H_s \mid J=2> = -1/2\, \lambda <r^{-1}> m^{-2}$$

$$<J=1 \mid H_s \mid J=1> = +1/2\, \lambda <r^{-1}> m^{-2}$$

$$<J=0 \mid H_s \mid J=0> = +1\, \lambda <r^{-1}> m^{-2} \quad (6\,b)$$

Then, it is easily seen that

$$R = \frac{E(2^{++})-E(1^{++})}{E(1^{++})-E(0^{++})} =$$

$$= \frac{\alpha_s <r^{-3}> \left[(3+4k)-3{,}5\,(1+k)^2\right] - \lambda <r^{-1}>}{\alpha_s <r^{-3}> \left[1/2\,(3+4k)+3/2\,(1+k)^2\right] - 1/2\,\lambda <r^{-1}>} \quad (6\,c)$$

Using for $<r^{-1}>$ and $<r^{-3}>$ the expressions developed in ref. [2], with the help of a variational method, and with the parameters

$$\lambda = 0.132 \text{ GeV}^2,\ m = 2 \text{ GeV},\ \alpha_s = 0.54,\ k = 1, \quad (7\,a)$$

we find $R=0.4$; as in ref. [1], this depends on our use of a 4-scalar confining potential.

We can go a little further and compute the absolute values of the P-wave and Ψ-η_c splittings. With the parameters ($7a$) and with the help of expressions ($6a$) and ($6b$) we get

$$E(1^{++})-E(0^{++}) = 100 \text{ MeV and } E(2^{++})-E(1^{++}) = 40 \text{ MeV} \quad (7\,b)$$

To determine Ψ-η_c we apply

$$E(1^{--})-E(0^{-+})=8\pi/3\,(1+k)^2\,\alpha_s\,|\phi(0)|^2 m^{-2}, \qquad (7c)$$

$|\phi(0)|^2$ being calculated from $\Gamma(\Psi\to e^+e^-)$ with the help of the Weisskopf–Van Royen formula.

Taking $\Gamma(\Psi\to e^+e^-)=5$ KeV, we have $|\phi(0)|^2=3.78\times10^{-2}$ GeV^{-3}, giving

$$E(1^{--})-E(0^{-+})=230 \text{ MeV}, \qquad (7d)$$

in approximate agreement with the experimental value.

Let us now see what we can say for the light–mesons. Here we have an additional difficulty due to the uncertainty in the position of the A_1 meson.

With the choice

$$\lambda=0.100 \text{ GeV}^2,\ m=0.34 \text{ GeV},\ \alpha_s=1\ 0\ ,\ k=0.5 \qquad (8a)$$

we have $R=(A_2-A_1)/(A_1-\delta)=0.4$. Taking $\delta(0^{++})$ at 0.970 GeV we get 1.15 GeV for the position of the A_1 (1^{++}) meson, and 1.190 GeV for the A_2 (2^{++}), which seems a bit low, its experimental position being at 1.31 GeV.

If for $|\phi(0)|^2$ we take the value quoted in ref. [6], $|\phi(0)|^2=$ $=2.9\times10^{-3}$ GeV^{-3}, we get the correct value for the $\rho-\pi$ splitting:

$$E(\rho)-E(\pi)=630 \text{ MeV}. \qquad (8b)$$

The value of α_s in $(8a)$ seems high but is in agreement with deep inelastic scattering data [5].

As for λ, its value is smaller than the value quoted in $(7a)$, but rememember these are only rough numerical estimates, intended as a guide to help us in carrying on the program defined in the introduction.

3—RESULTS FOR CHARMONIUM

We now present the results for charmonium obtained by numerically integrating equation (1) with the potential described by expressions (2) and (3). (Due in part to computing difficulties we cannot yet present results for the light–mesons and for the Ψ''_D, in the spectrum of charmonium).

With the following typical set of parameters

$$\lambda = 0.194 \text{ GeV}^2, \ \alpha_s(1.0) = 0.30, \ m_c = 1.6 \text{ GeV}, \ k = 0.73 \quad (9a)$$

(a=1.8 Fermi, C=0.9 GeV), we got for the Ψ - η_c splitting the value 280 MeV, and for the Ψ'-η'_c splitting 240 MeV. This last result is very interesting, because the η'_c comes close to 3.45 GeV, suggesting its identification with the state discovered at this energy.

The P-wave states come at

$$\chi(0^{++}) = 3.40 \text{ GeV}, \ \chi(1^{++}) = 3.49 \text{ GeV}, \ \chi(2^{++}) = 3.53 \text{ GeV}, \quad (9b)$$

giving R$\simeq$0.45.

It is important to notice that the value for α_s (1.0) used here is very small in apparent disagreement with the deep inelastic scatering data [5], but is in agreement with the initial asymptotic freedom expectations.

4 — CONCLUSIONS

In the preceding sections we saw how the large ground-state hyperfine splitting in the charmonium spectrum could be explained by an effective quark-gluon anomalous magnetic moment, in the context of a relativistic quark model previously developed. We assumed the anomalous moment to come from the short distance part of our potential (2), while keeping the long range confining potential as a 4-scalar; this was essential to get the correct value for the ratio

$$R = \{E(2^{++}) - E(1^{++})\} / \{E(1^{++}) - E(0^{++})\}.$$

Simple numerical estimates suggest that the same improvement in our model will also enable us to explain the spectrum of the light--mesons, especially the large ρ-π splitting.

However, and going back to charmonium, we must remember that the large hyperfine splitting is only half the problem, the other half being the fact that the magnetic dipole transition rate $\Gamma(\Psi \to \eta_c + \gamma) \leq 1$ KeV; any real solution will have to solve both the problem posed by the large Ψ-η_c splitting and by the small decay rate for $\Psi \to \eta_c + \gamma$.

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