CALCULATION OF THE IONIC TEMPERATURE OF AN ARGON PLASMA (*)

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ABSTRACT — Considering the experimental difficulty in determining the ion temperature of a plasma, namely at high pressures or high temperatures, this work aims at a theoretical determination of the ionic temperature (T_i) of an Argon plasma in a supposedly homogeneous region where the distribution functions of the electrons (e), ions (i) and neutrals (o) are assumed to be Maxwellian.

T_e (the electron temperature), T_o (the neutral temperature), n_o (the neutral density) and n_e (the electron density) are known. All temperatures are below 10 eV. It is also assumed that ions possess no specific heating mechanisms.

It is observed that for the same T_e, T_i approaches T_o for higher T_o. As a general rule, for higher T_e values, T_i values (at the same T_o) are found to be lower. Finally, a graphic representation of $\Delta_r = (T_i - T_o)/T_o$ as a function of T_o, for different T_e values, shows curves to intersect; this is due to the fact that the curve representing the ion heating rate at the same T_i is not a monotonous function of T_e, which shows that for a given T_i value there will always exist pairs of T_e values leading to the same heating rate value.

The results obtained can only be applied to large dimension plasmas with small applied fields. The validity conditions are close to those of a high pressure arc.

1 — INTRODUCTION

If it is relatively easy to obtain, experimentally, the electronic temperature of a plasma, the same cannot be said about the ionic temperature; in fact, there exists only one simple experimental technique which makes it possible to obtain it and wich consists in measuring the broadening due to the Doppler effect produced by

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thermal agitation, observable in certain particularly adequate lines. This technique meets with difficulties, both for high pressures, where the pressure broadening is generally dominant and for the high temperatures for which lines are to be found on the low wave lengths where it is rather difficult to obtain a sufficient resolution in the domain of wave lengths.

From this difficulty has arisen the necessity of a theoretical calculation of the ionic temperature.

Our purpose is, therefore, to calculate the ionic temperature of an Argon plasma in a supposedly homogeneous region where the distribution functions of the particles — electrons (e), ions (i) and neutrals — are assumed to be Maxwellian.

T_e (the electron temperature), and T_o (the neutral temperature) are known, as well as the neutral gas pressure and the plasma density (density of charged particles). This density varies between 10^{18} and 10^{20} m^{-3}. All temperatures are assumed to be below 10 eV.

It is also assumed that ions possess no specific heating mechanisms (magnetic field, ionic wave – instability).

To calculate the ionic temperature we shall start from the relation which expresses the stationary state characterized by the balance between the mean energy transmitted by electrons to ions through elastic collision, and the mean energy transmitted by ions to neutrals during their interaction processes.

2 — ENERGY BALANCE EQUATION REFERRED TO IONS

Ions are heated by collision with electrons and cooled by collision with neutrals. We shall use the same general approach for ion-electron and ion-neutral collisions even if there is a more exact method for the calculation of the former [2].

2.1 — *Ion heating rate by collision with electrons*

From Fokker — Planck term there results

$$\left[\frac{dE_i}{dt} \right]_{e-i} = n_e \, \bar{\nu}_{ei} \, f_{ei} \, E_e \tag{1}$$

$\bar{\nu}_{ei}$ – mean frequency of electron-ion collisions
f_{ei} – electron energy fraction which is, on the average, transferred to the ion in an elastic collision
n_e – electron density
E_e – electron mean energy

Cravath [1] established that the energy fraction which is, on the average, transferred in an elastic collision between particles with Maxwellian distributions from the faster particle (j) to the slower (k) is given by

$$f_{jk} = \frac{8}{3} \frac{m_j \, m_k}{(m_j + m_k)^2} \left(1 - \frac{T_k}{T_j}\right) \tag{2}$$

m_j – mass of particle j
m_k – mass of particle k

In the present case, particle j is the electron and particle k the ion. On the other hand, since $m_e << m_i$, $m_e + m_i \simeq m_i$.
Substituting in (2) we have

$$f_{ei} = \frac{8}{3} \frac{m_e}{m_i} \left(1 - \frac{T_i}{T_e}\right) \tag{3}$$

The collision mean frequency is given by

$$\bar{\nu}_{jk} = n_k \, \bar{\sigma}_{jk} \, \bar{g}_{jk} \tag{4}$$

$\bar{\sigma}_{jk}$ – mean cross section of kinetic energy transference
$\bar{g}_{jk}$ – mean relative velocity for Maxwellian distributions [1]

$$\bar{g}_{jk} = \left(\frac{8k \, T_j}{\pi m_j}\right)^{1/2} \left(\frac{m_j}{m_k} \frac{T_k}{T_j} + 1\right)^{1/2} \tag{5}$$

Introducing the simplification due to mass relations (for the Argon case $m_e / m_i = 1{,}36 \times 10^{-5}$), we have

$$\bar{g}_{ei} = \left(\frac{8k \, T_e}{\pi m_e}\right)^{1/2} \tag{6}$$

The mean cross section of an electron-ion elastic collision is approximately [2]

$$\bar{\sigma}_{ei} = \frac{q_e^2 \, q_i^2 \, ln \Lambda}{4 \pi \varepsilon_o^2 (3 k T_e)^2} \tag{7}$$

where $ln \Lambda$ is the so-called «Coulombian Logarithm».

$$\Lambda = 12 \pi n_e^{-1/2} (\varepsilon_o k T_e / q_e^2)^{3/2} \tag{7'}$$

As the first ionization potentials of the Argon atom are 15,75 eV and 43,37 eV, and since in our case all temperatures are below 10 eV, pratically all ions are simply ionized. Should it not be so, we would have to define the ionic mean charge:

$$\bar{q}_i = \frac{\Sigma_j \, n_{ij} \, q_{ij}}{\Sigma_j \, n_{ij}} \tag{8}$$

In our case, as previously seen, $q_i \simeq q_e$ and therefore (7) becomes

$$\bar{\sigma}_{ei} = \frac{q_e^4 \, ln \Lambda}{36 \pi \varepsilon_o^2 \, (k \, T_e)^2} \tag{9}$$

Substituting in (1) expressions (3), (4), (6), and (7), the following equation is obtained (*)

$$\left[\frac{d E_i}{dt} \right]_{e-i} = \frac{4 \, q_e^4 \, n_e \, n_i \, m_e \, ln \Lambda \, (1 - T_i / T_e)}{9 \pi \, \varepsilon_o^2 \, (2 \pi m_e \, k T_e)^{1/2} \, m_i} \tag{10}$$

2.2 — *Ion cooling rate by collision with neutrals*

$$\left[\frac{d E_i}{dt} \right]_{i-o} = - n_i \, \bar{\nu}_{io} \, f_{io} \, E_i \tag{11}$$

$\bar{\nu}_{io}$ — ion-neutral collision mean frequency
f_{io} — fraction of ion kinetic energy transferred, on the average, to neutrals, during collision
E_i — ion mean kinetic energy.

(*) A more exact description gives a factor 1/2 (instead of 4/9), which we shall use in the numerical calculations.

To calculate this term it is necessary to take into account the charge transfer collision phenomenon which consists in the capture, by the ion, of the peripheric electron of the neutral, and which results in the identity transfer of the particles.

A priori, the following cases may be considered as possible:

a) Purely elastic dominant collisions (because less frequent, charge transfer inelastic collisions are negligible)

$$A\,(W_A) + B^+\,(W_B) \rightarrow A\,(W'_A) + B^+\,(W'_B)$$

b) If charge transfer inelastic collisions are not negligible, two hypotheses have to be considered:

b.1) During the charge transfer collision there is no energy transference between particles beyond the one resulting from this change

$$A\,(W_A) + B^+\,(W_B) \rightarrow A^+\,(W_A) + B\,(W_B)$$

b.2) Besides the charge transfer there exists kinetic energy transference between particles

$$A\,(W_A) + B^+\,(W_B) \rightarrow A^+\,(W'_A) + B\,(W'_B)$$

Let us analyse the more general case b.2).

E will represent the energy before the collision and E′ the energy after the collision. $f_{io_{t.c.}}$ will be the kinetic energy mean fraction elastically transferred during the charge transfer collision.

Ion:

$$E'_i = E_o + f_{io_{t.c.}}\,E_i \qquad (12)$$

Neutral:

$$E'_o = E_i - f_{io_{t.c.}}\,E_i \qquad (13)$$

The kinetic energy lost, on an average, by the ion through collision with a neutral is given by:

$$\Delta E_i = E_i - E'_i = E_i - E_o - f_{io_{t.c.}}\,E_i \qquad (14)$$

Let $f_{io_{eq}}$ represent the ion energy fraction which is, on an average, transferred to the neutral during a charge transfer collision. Then,

$$f_{io_{eq}} = \frac{\Delta E_i}{E_i} = 1 - \frac{E_o}{E_i} - f_{io_{t.c.}} = \left(1 - \frac{T_o}{T_i}\right) - f_{io_{t.c.}} \qquad (15)$$

Let us now find the value of $f_{io_{eq}}$ for the other cases.

a) In this limit, since the charge transfer is negligible, $f_{io_{eq}} = f_{io}$ (kinetic energy fraction transferred on an average in an *i-o* purely elastic collision); f_{io} is determined by expression (2) where $m_o \simeq m_i$

$$f_{io} = 2/3\ (1 - T_o / T_i) \qquad (16)$$

b) In the limit b.1), $f_{io_{t.c}} = 0$

Then $$\Delta E_i = E_i - E_o$$

$$f_{io_{eq}} = (1 - T_o / T_i) \qquad (17)$$

Assuming that in expression (15) $f_{io_{t.c}} \leqslant f_{io}$, we can have for any of the cases considered

$$f_{io_{eq}} = \alpha\,(1 - T_o / T_i) \qquad (18)$$

where α varies from $1/3$ (case b.2, with $f_{io_{t.c}} = f_{io}$) to the unit (case b.1):

$$1/3 \leqslant \alpha \leqslant 1 \qquad (19)$$

The values of the elastic collision cross sections and of the charge transfer for the case of Ar at low energy being of the same order of magnitude [3], it seems correct to take an average value $\bar{\alpha} = 2/3$ and the total cross section (elastic + charge transfer). In the absence of experimental results on the energy band lower than 1 eV, the value of the total cross section should be obtained by extrapolation of the values available. It should be pointed out that, for extremely low collision energies this extrapolation is not valid, as in this case a polar binary complex may be formed, there arising then a situation which, for such energies, leads to a theoretically higher cross section [4].

Let us now determine $\bar{\nu}_{io}$

$$\bar{\nu}_{io} = n_o \; \bar{\sigma}_{io} \; \bar{g}_{io} \tag{20}$$

The experimental curve of $(\bar{\sigma}_{io})_{\text{t.c.}}$ as a function of E_i [5] is very approximately described by the expression

$$(\bar{\sigma}_{io})_{\text{t.c.}} = (6{,}9 - 0{,}25 \; ln \; E_i)^2 \times 10^{-20} \; \text{m}^2 \tag{21}$$

with E_i in eV.

$\bar{g}_{io}$ is obtained from expression (5) with $m_o \simeq m_i$,

$$\bar{g}_{io} = \left(\frac{8k\,T_i}{\pi m_i}\right)^{1/2} \left(1 + \frac{T_o}{T_i}\right)^{1/2} \tag{22}$$

Substituting in (11), and with $E_i = 3/2 \; k \; T_i$, we have

$$\left[\frac{dE_i}{dt}\right]_{i-o} = - n_i \; n_o \; \bar{\sigma}_{io} \left(\frac{8k\,T_i}{\pi m_i}\right)^{1/2} \left(1 + \frac{T_o}{T_i}\right)^{1/2} \left(1 - \frac{T_o}{T_i}\right) k \; T_i \tag{23}$$

3 — FINAL EQUATION

From equations (10) and (23) we get

$$\frac{T_e^{\,3/2} \; (T_i + T_o)^{1/2} \; (T_i - T_o)}{(T_e - T_i)} = \frac{n_e}{n_o} \; \frac{q_e^4 \qquad ln \, \Lambda}{8 \pi \, \varepsilon_o^2 \; (m_i / m_e)^{1/2} \; \bar{\sigma}_{io} \; k^2} \tag{24}$$

The solution of equation (24) to determine T_i is very complex since $\bar{\sigma}_{io} = f \; (T_i)$. On the other hand $ln \, \Lambda = f \; (n_e, \; T_e)$. However, from the observation of the dependence of $\bar{\sigma}_{io}$ on E_i and of the values of $ln\,\Lambda$ the following simplifying hypothesis may be considered:

Equation (21) was determined for E_i values between 1 eV and 100 eV (1 eV $\simeq$ 10.000 K). Since $T_e > T_i > T_o$, and considering the range of values used for T_e, for T_o and for n_e/n_o, it can be concluded that T_i will take values between the approximate limits of 300 K (0,03 eV) and 10 000 K (1 eV). Thus, and since $(\bar{\sigma}_{io})_{t.c.}$ varies very little with E_i, the curve for $E_i < 1$ eV can be extrapolated and its mean value taken in the above mentioned range:

$$(\bar{\sigma}_{io})_{tc} = 51 \times 10^{-20}\ \mathrm{m}^2$$
$$(\bar{\sigma}_{io}) = 102 \times 10^{-20}\ \mathrm{m}^2 \qquad (25)$$

Substituting in equation (24) the values of constants and the value of $\bar{\sigma}_{io}$ given by (25), we have

$$\frac{T_e^{3/2}\,(T_i + T_o)^{1/2}\,(T_i - T_o)}{(T_e - T_i)} = \delta \qquad (26)$$

and

$$T_e^3\,T_i^3 - (\delta^2 + T_e^3\,T_o)\,T_i^2 + (2\delta^2 - T_e^3\,T_o^2)\,T_i + (T_e^3\,T_o^3 - \delta^2\,T_e^2) = 0 \qquad (27)$$

where $\delta = 6.36 \times 10^6\,(n_e/n_o)^2\,ln\,\Lambda$

and $ln\,\Lambda$ is given in (7′).

Solution was worked out by a numerical method, results having been obtained for several electronic temperatures and n_e/n_o ratios. Two groups of typical curves are presented (Figs. 1 and 2).

4 — DISCUSSION OF RESULTS

It can be seen from Figs. 1 and 2 that for constant T_e, T_i approaches T_o for higher T_o. This result is to be expected since when T_o approaches T_e, the equilibrium temperature T_i which results from the energy balance equation

$$\left[\frac{d E_i}{d t}\right]_{e-i} = -\left[\frac{d E_i}{d t}\right]_{i-o}$$

tends to approach T_o and T_e.

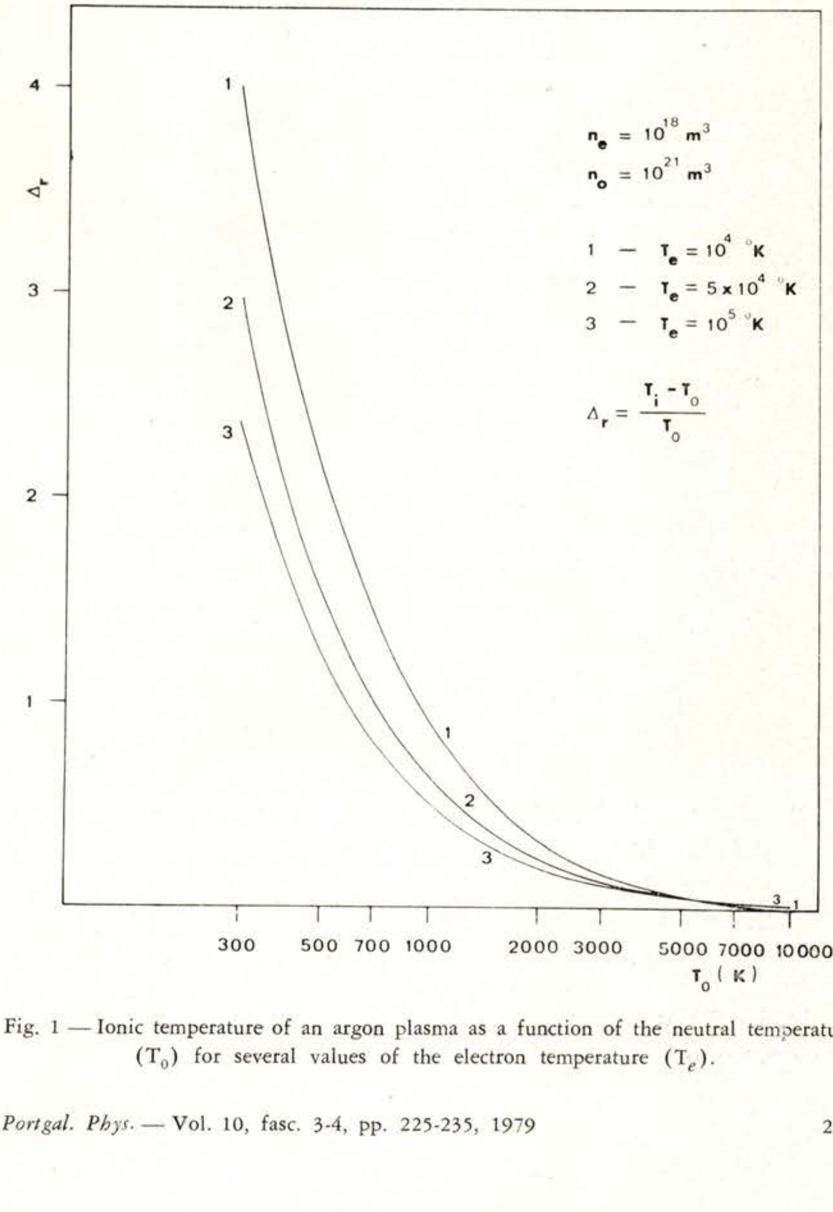

Fig. 1 — Ionic temperature of an argon plasma as a function of the neutral temperature (T_0) for several values of the electron temperature (T_e).

On the other hand and as a general rule, for higher T_e values we get lower T_i values (for the same T_o). However curves of Fig. 1 and 2 are found to intersect. This is due to the fact that the curve representing the ion heating rate for a given T_i is not a monotonous function of T_e. Thus there will always exist for a given T_i value, pairs of T_e values which lead to the same heating rate value.

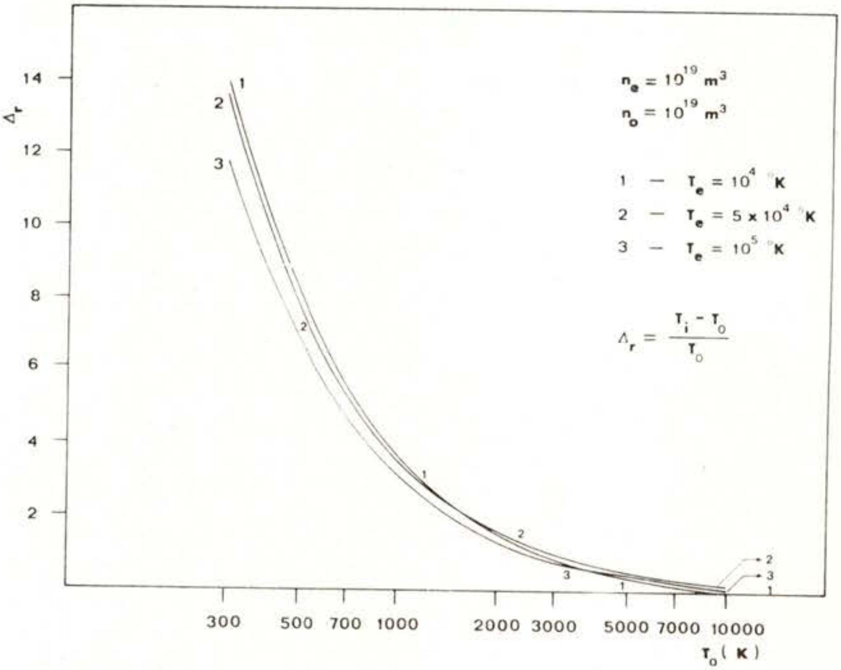

Fig. 2 — Ionic temperature of an argon plasma as a function of the neutral temperature (T_0) for several values of the electron temperature (T_e).

The results obtained have a limited field of validity since they can only be applied to regions of homogeneous plasma where the ion heating and cooling mechanisms are exclusively due to elastic collisions, respectively with electrons and neutrals. Now this can only occur for large dimension plasmas with small applied fields, whether electric or magnetic.

The validity conditions required are acceptable for a high pressure arc, the present results being then applicable.

On the other hand the parameters used in the equations can hardly be regarded as independent in a real plasma. A possible development of this work would be the inclusion of a balance condition for the neutrals with a view to a more consistent theory.

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