AN AUTOMATIC FARADAY TYPE MAGNETOMETER WITH A WIDE RANGE SENSITIVITY

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ABSTRACT — A new type of force magnetometer was designed and built. It can detect forces down to 10^{-4} dynes and the design enables measurement of weak and strong magnetic moments, with a resolution better than one part in 10^5. This was attained by keeping the sample in a static position by means of a feedback system. The balance is controlled by a minicomputer which allows the determination of a few thousands of data points per run; it is, thus, possible to test the data by statistical methods.

1 — INTRODUCTION

Methods for measuring the magnetisation of magnetic materials can be divided in two categories [1]:

i) force methods wherein one measures the magnetic force exerted on a sample placed in an inhomogeneous magnetic field;

ii) induction methods wherein one measures the signal induced in a detecting coil by the changing magnetic moment of the sample.

The Faraday method is an example of the first category. Due to its high sensitivity in measuring paramagnetic and diamagnetic susceptibilities it is a convenient method when dealing with small samples. Packing errors are nonexistent, since this method measures magnetic moments directly.

The principal difficulty associated with this method is the determination of the field gradient. If, as is usually done, this is

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produced by specially shaped pole pieces, the gradient is a complicated function and not easily determined with high accuracy. If the field gradient is produced by coils or current strips, it is usually very small and so the sensitivity is small. Moreover, it cannot be assumed that the field gradient will be independent of the permeability of the pole tips, i.e., of the field produced by the electromagnet.

The second category is exemplified by the vibrating sample magnetometer. Although this method has a lower sensitivity than the Faraday method it has the advantage of having an output which is directly proportional to the sample magnetisation at all fields.

The classical Faraday method does not stand as a convenient absolute method [2]; the limitation arises from the variation in the force due to the profile of the magnetic field when the specimen displacement takes place [3], which makes the field and field gradient determinations not very accurate. A variety of magnetometers based on different methods have been reported [4], [5], [6].

In the type presented in this work the sample is always at the same position due to an almost instantaneous restoring force, acting on the sample by a feedback system. A further problem associated with this method is that with a horizontal field it is impossible to have a vertical force without having a horizontal one and that any horizontal stiffness needed to overcome the horizontal force, inevitably decreases the sensitivity of the system to vertical forces.

This problem was solved in our magnetometer by deliberately making the system stiff in both directions and then using an extremely sensitive method for sensing the vertical force. A change on sample position can be detected with a sensitivity of 3×10^{-8} mm. The lateral motion is prevented by flat spirals made out of phosphor bronze (0.15 mm thickness).

2 — ELECTRONIC DESIGN OF THE DISPLACEMENT TRANSDUCER

Lion [7] and Neubert [8] discussed many of the factors affecting the choice of an electromechanical transducer for slowly varying displacements. Some more obvious features of a capacitive transducer are:

a) The forces exerted on the moving member by the measuring apparatus are electrostatic ones and these can be made small enough for most purposes.

b) The system responds to the average displacement of a large area of a moving member.

c) Since the resistance of practical conductors can be taken as zero in the present context the performance is determined almost wholly by the geometry of the transducer, and this can be made fairly simple.

The capacitance between two isolated electrodes of simple geometry can usually be written, at least approximately, in the form of an area divided by a separation.

For an isolated pair of nearly parallel plates one larger than the other, the capacitance is very sensitive to a change in the average separation of the plates, and very insensitive to any other relative motion.

Let A be the area of the smaller plate, and ε the permittivity of the medium between the plates. Suppose that when the separation is changed from X_0 to $(X_0 - X)$, the capacitante changes from C_0 to C_1. Then, neglecting edge effects [9],

$$(C_1 - C_0)\,(1 - X/X_0) = \varepsilon\, A\, X/X_0^2 = C_0 X/X_0 \qquad (1)$$

A particular convenient arrangement uses three nearly equally spaced parallel plates, the outer two being a fixed distance apart. A displacement of the middle plate causes one of the two capacitances so formed, say C_1, to increase and the other, C_2, to decrease. If the separation of the plates are respectively $(X_0 - X)$ and $(X_0 + X)$, and C_0 is the value of C_1 and C_2 when X is zero then

$$(C_1 - C_2)\,(1 - X^2/X_0^2) = 2\,\varepsilon\, AX/X_0^2 = 2C_0 X/X_0 \qquad (2)$$

The arrangement has thus the additional advantages of:

i) doubling the output

ii) reducing the nonlinearity

iii) making the electrostatic force on the centre plate zero, when the system is exactly balanced.

Departures from linearity are predictable and stable. There is continuous and smooth change in output against displacement. The geometry of the transducer is more easily calculated than with magnetic devices because there is no penetration of the electrostatic field in the metal electrodes. Sensitivity is determined mainly by the external circuit, and with a good design a resolution of 1 part in 10^7 or better is possible. Stability is mostly a matter of mechanical design, although dielectric losses may cause some variation.

Fig. 1 shows a circuit of a differential capacitive transducer with an inductive divider connected to form an a.c. bridge circuit.

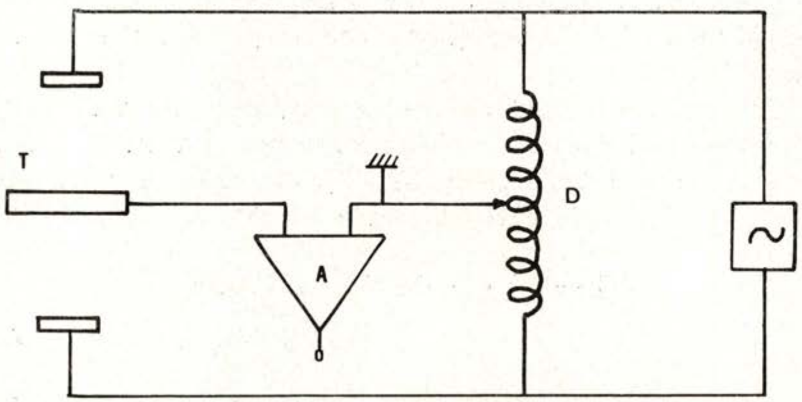

Fig. 1 — The A.C. bridge circuit.

The carrier signal C energizes both the inductive divider D and the transducer T. Since the transducer draws very little current the impedance of the leads to it is not significant. Also the inductive divider has low output impedance, and so earth admittances cause little error. The output impedance of the transducer is high and needs a suitable amplifier to retain as much of the output signal as possible. The carrier phase and amplitude is detected by a phase-sensitive detector so that the sense and amplitude of the displacement is indicated. This circuit may have an analogue output which indicates the 'out of balance' signal between the halves of the bridge.

The differential capacitive sensor has an impedance of about 1 pF between each plate. There is a large shunt impedance due to the input cable capacities of the order of 1,000 pF. However, because the input amplifier is a 'virtual ground amplifier', (Fig. 2),

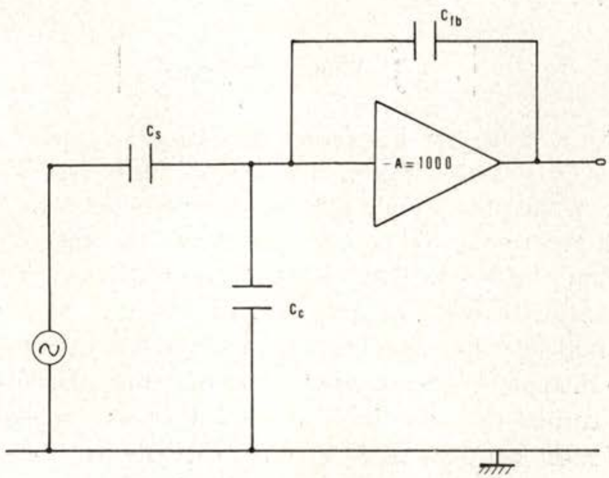

Fig. 2 — Input amplifier used as a 'virtual ground amplifier'.

then the effect of the shunt capacity can be greatly reduced. Considering the circuit of Fig. 2. the gain to signal will be C_s/C_{fb}. With the given values $C_s = 1$ pf and $C_{fb} = 10$ pF; this gives a gain of 1/10 to the signal. However if the input capacitance were to be measured it would be about 10,000 pF, as the amplifier acts as a capacitance multiplier on C_{fb}. Now if this is shunted by the 1,000 pF cable capacitance it will only give rise to a 10 % fall in gain to signals via C_s.

The signal to noise ratio gets progressively worse as the input shunt capacity increases. If it is necessary to make very high resolution measurements, care must be taken to see that the cable capacity is kept to a minimum.

The accurate measurement of a.c. signals is often affected by the presence of noise. The noise may often be reduced by special screening and earthing arrangements. However these are sometimes inappropriate and usually quite difficult to apply. In any case they do nothing to reduce random noise, the limiting factor to resolution in measurement of small signals.

The output of the phase-sensitive detector (in this work) is a d.c. signal on which a double frequency carrier signal (ripple) and wideband noise are superimposed; these spurious signals can be almost totally removed by filtering in a later stage. The d.c. output level is given by

$$\left| \left(\frac{1}{\pi} \int_0^{\pi} A \sin x \, dx \right) \cos \phi \right|$$

The term inside the brackets is the average value of a rectified sine wave over half a cycle. The cos ϕ term comes from the difference in the phase angle between the detector drive waveform and the input signal. When the angle ϕ is 90° then there is no output from the p.s.d.. This means that the leakage resistance of the transducer will not give rise to any d.c. output, and an accurate null can still be obtained. This is one major advantage of the p.s.d. type of circuit. Another is that the signal component has been turned to a d.c. level, and as a direct consequence the wideband noise can now be filtered by a simple RC low pass filter.

In reality, the plates of the transducer are not isolated. If the conductors in the neighbourhood of the plates are not set to definite potentials (i.e. are left electrically floating), they can have a large effect on any capacitance measurement, and it is essential to connect them to a common terminal which will most conveniently be earthed. These conductors must therefore remain as far as possible fixed relative to the plates. The necessarily flexible leads to the plates must be arranged with care, but it is much easier to avoid stray capacitance between two leads than between one lead and earth.

It should be mentioned at this stage some of the most important mechanical disturbances likely to occur [10].

Temperature variations are a serious factor originating expansion of the components in a degree which cannot be neglected. Also temperature-time variations can be of importance even in devices made out entirely of the same material as thinner parts will respond faster than thicker ones to external temperature changes.

In this circumstances a symmetrical design and a thermostatically controlled enclosure are required. Also the instrument should be situated in a massive evacuated metal box, its conducting walls providing an adequate electrostatic insulation, important mainly at frequencies at which the mechanical components resonate, i.e. 50 Hz and above. Low frequency vibrations can also cause disturbances if they make the instrument platform tilt, since this may change the relative direction of gravity and the elastic forces in the structure.

3 — DESCRIPTION AND PERFORMANCE OF THE MAGNETOMETER

The conditions of the design of the type of magnetometer presented here give rise to the following characteristics:

1. It is based on the classical Faraday method but having the advantage of a feedback system restoring the sample position (see Fig. 3).
2. Forces as small as 1×10^{-4} dynes can be detected.
3. Noise at output — 0.1 mV r.m.s.
4. Signal-to-noise ratio — better than 5.0.
5. Precision — up to 1.5×10^{-6} g $<>$ 1×10^{-4} dynes.
6. Resolution — 1 part in 10^5.

Fig. 3 shows schematically a sketch of the magnetometer and the associated electronic circuits. The output in the form of a single ended voltage across a standard resistor can be fed into three devices simultaneously: a X, T/Y recorder, a digital voltmeter and a minicomputer with two simultaneous outputs (punched paper tape and printed data onto a teletype).

4 — CALIBRATION AND FIELD GRADIENT

Although it is possible, in principle, to measure the field gradient absolutely, it is difficult to attain an accuracy much better than about 0.1 %, so the field gradient was determined, in this work, as a function of field using samples of known magnetisation.

It is, however, very clear that the accuracy of the final measurements can be no greater than the accuracy with which the field gradient can be determined. This, in turn, is limited by the accuracy with which the magnetisation of the sample used for calibration is known. There are two somewhat different approaches to the problem. In the first place one can use a paramagnetic sample. The magnetic moment of an ideal paramagnet is directly proportional to the field so that the field gradient is determined in terms of the field and the susceptibility.

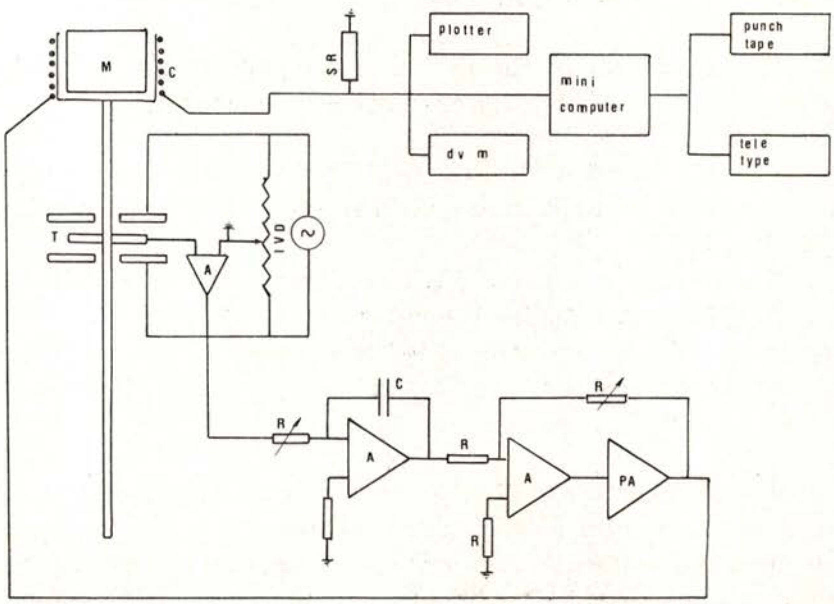

Fig. 3 — Sketch of the magnetometer and the corresponding logging system.

In practice ideal paramagnets have a rather low susceptibility, the forces to be measured would be rather small and the errors involved correspondingly high. To overcome the latter problem one can select a real material with a large paramagnetic susceptibility but in this case there is a risk that its susceptibility might be field dependent. Our solution to this problem was to use the high susceptibility of a rare earth metal at room temperature and to establish the field-independence of its susceptibility by an independent experiment.

Alternatively one can use a ferromagnetic sample, the advantage here being that the forces to be measured are large and the field does not have to be measured. Instead, the field-dependence of its ferromagnetic moment must be known, either from a previous, independent measurement or by other means. This is essentially the method of Aldred et al. [11]. They used a sample of pure iron and noted that, according to simple spin wave theory and using values of the spin wave stiffness obtained from inelastic neutron scattering, the intrinsic magnetisation of iron at 5.3 K would be independent of field (up to 20 kOe) to within 0.001 %. They then assumed that would be true in practice and used this constant value of the magnetisation of iron at 5.3 K to determine the field gradient.

The paramagnetic samples used for the calibration were single crystals of Tb, Dy and Ho. In order to test the field independence of these samples, susceptibility measurements were performed using a constant field gradient (by means of Helmholtz coils). These measurements were made in applied fields up to 80 kOe.

In this particular experiment the magnetometer output is proportional to the magnetisation values. That is, at each value of the field the ratio output/H (applied field) is proportional to χ. Thus a study of how this ratio varies with the field will establish the degree of field independence of the susceptibility. Table 1 shows, for the Tb sample, magnetised along its easy axis at room temperature, values of output/H as a function of H. The value of the standard deviation obtained is 2×10^{-5} and thus the field independence of χ is established within 0.001 %.

Using the magnetometer described in this work three runs for each sample were performed at room temperature, along the easy axis. The resulting field gradients thus obtained were in good agreement within 0.001 %. The averaged field gradient was used in all measurements of this work.

The field gradient was also determined using a sample of annealed polycrystalline iron at 5.3 K. Because of its polycrystalline nature, the saturation magnetisation is reached asymp-

TABLE 1 — Magnetometer Output as a function of H

Applied field H (k0e)	Force × const. (volt)	Force × const/H (volt/k0e)
10	19.1426	1.91426
20	38.2859	1.91429
30	57.4289	1.91429
40	76.5712	1.91428
50	95.7144	1.91429
60	114.856	1.91426
70	133.998	1.91427
80	153.144	1.91430

totically, the departure from saturation being represented by the second term in the equation [11]

$$\sigma(H, 5.3) = \sigma(\infty, 5.3) - 0.07619\,\alpha K_1^2 [\sigma(\infty, 5.3)\rho^2 H_i^2]^{-1} + \chi H_i \quad (3)$$

with $\sigma(\infty, 5.3) = 221.71$ emu/g, $K_1 = 5.25 \times 10^5$ erg cm^{-3} and $\rho = 7.85$ g cm^{-3}, where $\sigma(\infty, T)$ is the saturation magnetisation at T(K), H_i is the internal field, $K_1(T)$ is the first anisotropy constant, ρ is the density and χ the intrinsic susceptibility. α is a numerical factor arising from interactions between the crystal grains [13]; in the field range 10 to 18 k0e it varies from 0.61 to 0.68.

Although this second term has a well-established theoretical basis it has never been put to a satisfactory test and the field-dependence of the factor α has never been verified by experiment. Fortunately the term accounts to no more than 8×10^{-3} e.m.u./g within the field range 10-18 k0e and the uncertainties in this correction are unlikely to be important. The intrinsic susceptibilty term is another matter. The experimentally determined values of χ for iron range from 4.14×10^{-6} to 5.46×10^{-6} emu/g.

For the lowest of these values χH changes 3.3×10^{-2} emu/g when H goes from 10 to 18 k0e and this clearly is important.

Let us consider briefly the effect of this term. Suppose the true field gradient is G and we measure the force F on a ferromagnetic sample for which

$$\sigma = \sigma_0 + \chi H = \sigma_0 (1 + \chi H/\sigma_0). \tag{4}$$

Then

$$F = k\, G\, \sigma \tag{5}$$

Suppose we overlook or otherwise ignore the χH term, then we shall determine a spurious field gradient G′ given by

$$F = k\, G' \sigma_0 \tag{6}$$

so that

$$G' = G\ (1 + \chi H/\sigma_0) \tag{7}$$

If we now use this spurious value G′ to determine the magnetisation σ_1 of another material for which the magnetisation is genuinely independent of the field then we shall measure a force

$$F = k\, G\, \sigma_1 \tag{8}$$

but we shall determine a spurious magnetisation σ_1' given by

$$F = k\, G'\, \sigma_1' \tag{9}$$

Clearly

$$\sigma_1'/\sigma_1 = (1 + \chi H/\sigma_0)^{-1} \tag{10}$$

and since $\chi H << \sigma_0$ this may be written to very good aproximation as

$$\sigma_1' = \sigma_1\ (1 - \chi H/\sigma_0) \tag{11}$$

or

$$\sigma_1' = \sigma_1 - \sigma_1 \chi H/\sigma_0 \tag{12}$$

To correct for this spurious field dependence all we need to do is to add a term $\sigma_1 \chi H/\sigma_0$ to all the values of the magnetisation. However, if we are principally concerned with analysing the field-dependence of the magnetisation this simply has the effect of adding a temperature-dependent susceptibility $\sigma_1 \chi/\sigma_0$. Since

χ is not known with certainty, for iron, the term may as well be omitted from equation (3). When a definite value for χ for iron is finally obtained all the results achieved through the use of a spurious value for G can be corrected using the simple procedure outlined above. It should be mentioned that, using polycrystalline iron as a calibration sample the field gradients were determined using the equations,

$$\sigma(H, 5.3) = \sigma(\infty, 5.3) - 0.07619 \; K_1^2 [\sigma(\infty, 5.3) \rho^2 H_i^2]^{-1} \quad (13)$$

and

$$\sigma(H, 5.3) = \sigma(\infty, 5.3) - 0.07619 \; K_1^2 [\sigma(\infty, 5.3) \rho^2 H_i^2]^{-1} + \chi H_i \quad (14)$$

It was found that field gradients calculated from equation (13) agreed better with those obtained from the paramagnetic samples than those calculated from equation (14) using $\chi = 5.46 \times 10^{-6}$.

The results are shown for comparison in Table 2.

TABLE 2 — Comparison of the field gradients

A/C	B/C	Internal field H_i (k0e)
1.000002	0.99941	5.6
1.000003	0.99896	10.7
1.000005	0.99833	18.1

A — field gradient derived from equation (13)
B — field gradient derived from equation (14)
C — field gradient derived from paramagnetic samples

This result, especially the closeness of the figures in the first column is very difficult to explain. We cannot rule out the possibility that all the measurements of χ for iron are incorrect and that the true value is much less than the accepted values. However the value of χ needed to reconcile the figures in column 1 is only $\sim 1.5 \times 10^{-8}$ and this poses great theoretical problems. The great weight of evidence is in favour of a susceptibility of about 4×10^{-6} in which case the use of equation (14) is certainly incorrect. The most likely explanation seems to be that the determination of the field gradient from paramagnetic samples involves a knowledge of the field. The minimum value in column 2 is only .99833 and this could be accounted for by an error in the characteristics (here taken to include reproducibility and accuracy of positioning) of the Hall probe used to measure the field. However it is surprising that the error acts in such a way as to cancel almost exactly the effect of the χH term in equation (14).

Three sources of systematic errors are present in a given run. The first is due to uncertainty in the force measured at zero field. This will produce systematic errors at all points of a particular isotherm. The second is due to possible temperature differences between the sample and the thermometer. The runs are not strictly isothermal and some temperature drift occurs with time. However the occurence of a temperature drift always leads to the possibility of a temperature lag between the thermometer and the sample and consequently to a systematic error in the temperature. The third source occurs when the calibration sample is removed from the apparatus and another sample introduced. Because of slight spacial variations in the field gradient, any uncertainty in positioning a sample would produce a systematic error in the magnetisation.

The use of small pole gaps in a conventional magnet produces a reaction, the so-called image effect between the specimen and the pole-pieces. This varies directly with the permeability of the pole faces and so depends on the state of saturation of the magnet.

In the present arrangement image effects are automatically included in the determination of the field gradient. There are grounds for believing that these effects are small. Several measurements were made, under identical conditions, of the forces on different specimens of the same material. The force per unit mass was constant to within 0.2 % for a considerable range of masses [12].

5 — FINAL REMARKS

Undoubtedly the advantages of the magnetometer described above are obvious and manifold, and only the most important points will be stressed and schematized here.

1. The great accuracy and sensitivity of the instrument is due to the high resolution of the sensor (differential capacitive transducer) allied to the high precision inductive voltage divider (with a division accuracy of 1 p.p.m.).

2. The instrument allows the investigations to be performed on a practically static sample position, which evidently permits a very precise measurement of the internal field.

3. The automatization of the system through its connection to a minicomputer gives the enormous advantage of the acquisition of a great number of data points up to 1,000 in a single run, where the applied field varies up to 20 k0e during a sweep of approximately 30 seconds.

The successive readings during the scanning of the applied field are done at well controlled steps by means of a convenient electronic triggering system designed for the purpose. In the present work it was found satisfactory the use of intervals of 35 0e between steps.

The applied field is read at static sample position by means of a Hall probe and the system trigger-minicomputer receives simultaneosly at each chosen step a pair of readings, one from the probe and the other from the magnetometer.

As a result the pair of readings field-force is a well defined one for each run at any time. This is a crucial advantage of this set up which will support the high accuracy of the relative measurements.

4. The fact that each run proceeds in a very short time (~ 30 s) assures that temperature fluctuations in the sample will be minimal. Also, it makes possible to obtain a very good average over a large number of repeated runs in a reasonable period of time.

5. Finally, perhaps the most important feature is the possibility of applying in a reliable manner powerful computer fitting procedures which are only possible over a large number of data

points. It is, therefore, quite clear that the high precision combined to the great number of points obtained are the most significant features of the magnetometer.

6 — SUMMARY AND CONCLUSIONS

The instrument constructed proved very successful and it has been used for several measurements [14], [15], [16]. The use of high-speed data acquisition coupled with the use of a minicomputer to process the data leads to a new level of precision of measurements of this kind which, coupled with the immense amount of data collected, enables one to investigate the excitations from the ground state of ferromagnetic materials, from the measured field dependence of the intrinsic magnetisation at various temperatures.

Moreover, these measurements can be performed, on suitable crystals, without the use of unusually high magnetic fields using an ordinary laboratory electromagnet.

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