NON-COLLINEAR BREAKING OF COLLINEAR SYMMETRIES AT FINITE MOMENTUM TRANSFER

J. PULIDO (*)

Department of Mathematical Physics. University of Birmingham, U. K.

(Received 4 August 1982; revised version 7 December 1982)

ABSTRACT — The breaking of the $SU(2)_w$ collinear symmetry arising when leaving the forward direction in scattering processes is studied. The starting point is the null-plane hydrogen atom Hamiltonian of Bell and Ruegg which exhibits this $SU(2)_w$ symmetry in an approximate form. Introducing an electromagnetic interaction with non-zero momentum transfer photons, it is found that breaking is mainly due to transversely polarized photons. Application to hadron-hadron scattering can only be realistic in the low q^2 exchange sector.

1 — INTRODUCTION

The null plane dynamics of hydrogen-like atoms was studied by Bell and Ruegg (BR) [1] in an approximation depending on c, the velocity of light, being large. Their ground state Hamiltonian exhibits an approximate $SU(2)_w$ symmetry which arises upon neglecting third order and higher terms in c^{-1} and which we will use as our starting point (1). They implicitly assume the interaction with $q = 0$ photons in their calculation of the magnetic moment. In this paper we examine the breaking process of a collinear symmetry like $SU(2)_w$ by calculating the interaction with non-zero momentum transfer ($t = q^2 \neq 0$) photons.

To this end we split the Hamiltonian into two terms

$$H_0 + H_I \tag{1.1}$$

(*) Present Address: Centro de Física da Matéria Condensada, Av. Prof. Gama Pinto 2, 1699 Lisboa Codex, Portugal.

(1) Either expression (17) or (26) of [1].

where H_0, the free part, given by BR, is supposed to be symmetry conserving and H_I, depending explicitly on q^2 through the form factors, will be symmetry breaking. The basis for the analysis of the symmetry breaking will be to compare the order of magnitude of the q^2 dependent terms involving spin in H_I with the terms of H_0.

The plan of the paper is as follows: in part 2 the calculation of the form factors of the hydrogen atom is outlined. In part 3 the $SU(2)_w$ symmetry breaking is analyzed in order to determine the minimum value of q^2 which breaks the symmetry. The result is valid for the long range sector of the potential.

2 — THE FORM FACTORS OF THE HYDROGEN ATOM

The model we use for the hydrogen atom is that of a bound state in a Coulomb-like potential. Thus the proton is just a force centre and the interacting external photon only «sees» the electron spin and electron charge. The hydrogen atom is therefore considered as a charged spin 1/2 particle with structure. The electromagnetic interaction of a non-point-like spin 1/2 particle is given by the usual expression [2, 3]:

$$\bar{\psi}\,[\,i\,\gamma_\mu\,e_\mu\,F_1(q^2)+\sigma_{\mu\nu}\,q_\nu\,e_\mu\,F_2(q^2)\,]\,\psi \qquad (2.1)$$

to which corresponds the diagram of Fig. 1

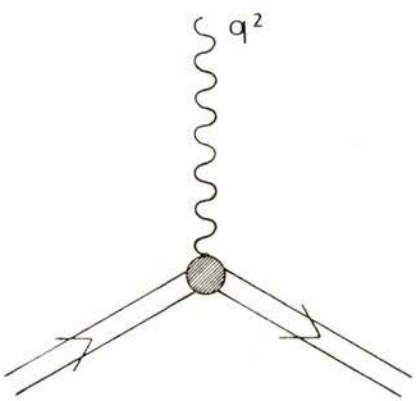

Fig. 1 — Photon-hydrogen atom interaction with a structure described by the form factors.

Here $F_1(q^2)$, $F_2(q^2)$ are the form factors of the hydrogen atom (electric and magnetic, respectively), q_ν is the photon four-momentum, e_μ is the polarization four-vector of the photon, $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]$; the Dirac matrices are (1)

$$\vec{\gamma} = \begin{bmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{bmatrix} \qquad \gamma_4 = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} \tag{2.2}$$

and $\bar{\psi} = \psi^+ \gamma_4$. Expression (2.1) is to be calculated between initial and final states which are just described by solutions of the free particle Dirac equation.

Here the information on the structure of the system is all contained in the form factors. In order to evaluate these, we equate the above description (2.1) for the interaction to the one within the impulse approximation in which the structure of the system is all contained in the initial and final wave functions and to which corresponds the diagram of Fig. 2.

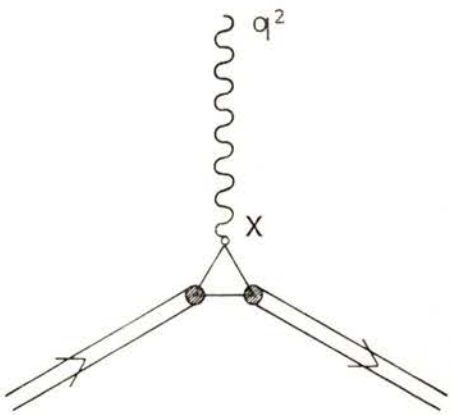

Fig. 2 — The same process considered as a point-like interaction. Here the structure lies in the wave functions of the hydrogen atom.

Here, contrarily to Fig. 1, we have now a point-like interaction at x of the usual form: $ie\gamma_\mu e_\mu$.

In the case of deuteron scattering, the amplitude for the diagram of Fig. 2 was written by Abers, Burkhardt, Teplitz and

(1) At this stage we use the conventional «low energy» Dirac notation.

Wilkin (ABTW) [4] (1) and becomes in our case after some manipulation [5] in which the convolution theorem was used:

$$\int d^3r\ \bar{\psi}_f(\vec{r})\ ie\gamma_\mu e_\mu \psi_i(\vec{r})\ \exp(i\vec{q}\cdot\vec{r}/2) \qquad (2.3)$$

where the ψ's are now electron spin wave functions within the hydrogen atom (see for instance [6]), $\vec{q}$ is the 3-momentum transfer carried by the photon and $\vec{r}$ is the distance of the electron from the proton.

In equating the two descriptions (2.1) and (2.3), care must be taken of the motion of the final state relative to the initial one. In the case of (2.1) this is explicitly included in the form of the spinor $\bar{\psi}$. As for (2.3), and assuming the initial state at rest we apply a Lorentz boost along the z-axis to the final wave function, in order to bring it to rest. This is given by [7]

$$S_3 = \exp(-\omega\,\sigma_{03}/2) \qquad (2.4)$$

where ω, the rapidity of the transformation, is:

$$\mathrm{ch}\ \omega = (1-\beta^2)^{-1/2}$$

where $\beta = Q/E_2$ is the velocity. Q is the z-component of the 3 momentum of the final state (assumed to be along the z-axis, $q = (\nu, 0, 0, Q)$), M its mass, and E_2 its energy [5]:

$$E_2 = (2M^2 - q^2)/2M^2,\ Q = (q^4 - 4M^2q^2)^{1/2}/2M,\ \nu = q^2/2M \qquad (2.5)$$

Separating transverse components (spin-flip) and longitudinal and time-like components (non-flip), we obtain the following two equations:

Transverse

$$\int d^3r\ \bar{\psi}_f(\vec{r})\ ie\,\vec{\gamma}_\perp\cdot\vec{e}_\perp\ e^{i(\vec{q}/2)\cdot\vec{r}}\ \psi_i(\vec{r})$$

$$= [0, 1, 0, Q/(E_2 + M)]\cdot\{\vec{\gamma}_\perp\cdot\vec{e}_\perp F_1 + \sigma_{\perp\nu} q_\nu e_\perp F_2\}\cdot \begin{bmatrix} 1\\0\\0\\0 \end{bmatrix}$$

(1) Equation (2.18) of ABTW.

Longitudinal and time-like

$$\int d^3r\ \bar{\tilde{\psi}}_f\ (\vec{r})\ ie\ (\gamma_0\ e_0 - \gamma_3\ e_3)\ e^{i\ (\vec{q}/2)\ \cdot\ \vec{r}}\ \psi_i\ (\vec{r})$$

$$= [1,\ 0,\ Q/(E_2 + M),\ 0]\ \cdot\ \{ i(\gamma_0 e_0 - \gamma_3 e_3) F_1 + (\sigma_{\nu_0} q_\nu e_0 - \sigma_{3\nu} q_\nu e_3) F_2 \} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where the twiddles over the final wave functions represent the Lorentz boost given by (2.4): $\tilde{\psi}_F = S_3\ \psi_f$.

Solving this system of equations, we obtain for the hydrogen atom form factors the following expressions:

$$F_1(q^2) = (q^2/2M)\ \ F_2(q^2) + (e/2M)\ z\ (1 + y^2)^{-2} \quad (2.6)$$

$$F_2(q^2) = e\ z^{-1}(1 + y^2)^{-2} + e/(\mu^2 + 1) \cdot z^{-1}(1 + y^2)^{-2} + \mu e a z/4M\ (\mu^2 + 1)\ \cdot\ (1 + y^2)^{-2} \quad (2.7)$$

$$- \mu^2\ e/(\mu^2 + 1)\ z^{-1}\ [(2 + y^{-2})(1 + y^2)^{-2} - y^{-3} \text{ang tg } y]$$

where

$z = (4M^2 - q^2)^{1/2}$, $y^2 = (a/8M)^2\ (q^4 - 4M^2q^2)$, $\mu^2 = (mc - E)/(mc + E)$, $e = 1/\sqrt{137}$ is the electron charge and a the typical electron orbit radius.

Therefore

$$F_1(0) = e\ ,\ F_2(0) = e(2 - \mu^2 + a\mu M)/2M(\mu^2 + 1) \simeq e/4 \quad (2.8)$$

$$F'_1(0) = (1/2M)\ F_2(0) + e(M^2a^2 - 1)/8M^2 \simeq ea^2/8 \quad (2.9)$$

$$F'_2(0) = e(1 + M^2a^2)/16\ M^3\ .\ [1 + 1/(\mu^2 + 1)] + \mu e a\ (a^2M^2 - 1)\ /\ 16\ M^2\ (\mu^2 + 1) \quad (2.10)$$

$$+ \mu^2 e\ /\ 160M^3\ (\mu^2 + 1)\ .\ (9a^2M^2 - 20) \simeq a^3 \mu e\ /\ 16\ (\mu^2 + 1)$$

These form factors are smoothly decreasing functions of q^2 behaving as $(q^2)^{-2}$ for large q^2. It is easy to check that $F_1(0) = e$

as expected, since the hydrogen atom in this model is a charged particle with charge e. This model also gives a reasonable prediction for the mean square radii of the spatial distributions [2, 5].

3 — THE ANALYSIS OF THE SYMMETRY BREAKING

Expanding the form factors in series of q^2, (2.1) can be converted into ($F_2(0) = \mu_M$ = magnetic moment):

$$\bar{u}\,[\gamma_\mu e_\mu (e + q^2 F'(0) + \ldots) + \sigma_{\mu\nu} q_\nu e_\mu (\mu_{..} + q^2 F_2'(0)]\,u \qquad (3.1)$$

This is of course a transition matrix element which, since we are treating the interaction in perturbation theory, is directly related to the Hamiltonian. However, it can only provide a quantitative measure of BR symmetry breaking if we use the usual «high energy» Dirac notation as in [1],

$$\vec{\gamma}_\perp = \begin{bmatrix} -\sigma_3 \vec{\sigma}_\perp & 0 \\ 0 & -\sigma_3 \vec{\sigma}_\perp \end{bmatrix}, \quad \gamma_3 = \begin{bmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{bmatrix}, \quad \gamma_0 = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix}$$

$$\bar{u} = u^+ \gamma_0 \quad , \quad \sigma_{\mu\nu} = i/2\,[\gamma_\mu, {}_\nu] \quad , \quad S_3 = \exp(-i\,\omega\,\sigma_{03}/2)$$

to which correspond the following solutions for the free Dirac equation

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad , \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

for spin + and — respectively.

As we said previously we will now compare the order of magnitude of the spin dependent terms arising in (3.1) with the

spin dependent terms of BR's free Hamiltonian for the ground state hydrogen atom in the null-plane, which is repeated here (1):

$$\begin{aligned}\hat{h} &= 1/2\ (\eta + \eta^{-1}) \\ &+ 1/2\ p^2_{\perp}\ \eta^{-1} + 1/2\ v + 1/2\ \eta^{-1}\ v\ \eta^{-1} \\ &+ 1/2\ \eta^{-1}\ [\sigma_3 \vec{\sigma}_{\perp} \cdot \vec{p}_{\perp}\ ,\ v\ ]\ \eta^{-1} \\ &+ 1/2\ \eta^{-1}\ \vec{\sigma}_{\perp} \cdot \vec{p}_{\perp}\ v\ \vec{\sigma}_{\perp} \cdot \vec{p}_{\perp}\ \eta^{-1} + 1/2\ \eta^{-1}\ v\ \eta^{-1}\ v\ \eta^{-1} \qquad (3.2) \\ &+ \ldots\end{aligned}$$

where p is the momentum of the hydrogen atom electron, supposed to be a first order small quantity, v the binding Coulomb potential and η denotes the z-differentiation in the null-plane. We note that the terms up to second order in small quantities are spin independent. Introducing $q^2 \neq 0$ photons via (3.1), the appearance of any spin dependent terms of order of magnitude equal or greater than p^2 will therefore break BR symmetry (2). To state it more precisely, a term like $(\vec{\sigma} \cdot \vec{p})\ p$ will be responsible for a symmetry breaking as well as something like $(\vec{\sigma} \cdot \vec{p})$ (even stronger effect), but $(\vec{\sigma} \cdot \vec{p})\ p^2$, for instance, will not.

We need to adopt units as BR, that is, $\hbar = c = m = 1$. Thus the Compton wavelength of the electron is our unit of length: $\hbar / mc = 1$.

From ref. [6], $a = 137$ (in units of $\hbar / mc$) and making use of the uncertainty principle:

$$p = 1/a = 7.3 \times 10^{-3}\ ,\ p^2 = 5.3 \times 10^{-5} \qquad (3.3)$$

Corresponding to the two amplitudes considered before, there are two types of symmetry breaking to consider: one for transverse photons and the other for longitudinal and time-like ones.

---

(1) Expression (17) of [1].

(2) $p^2 = \vec{p} \cdot \vec{p}$.

3.1 — *Symmetry breaking for transverse photons*

The interaction Hamiltonian assumes in this case the form

$$\overline{S_3 u_f} \left\{ A \begin{bmatrix} 0 & -\sqrt{2}\,e_- & 0 & 0 \\ \sqrt{2}\,e_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{2}\,e_- \\ 0 & 0 & \sqrt{2}\,e_+ & 0 \end{bmatrix} \right. \tag{3.4}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & -\sqrt{2}\,e_- \\ 0 & 0 & -\sqrt{2}\,e_+ & 0 \\ 0 & \sqrt{2}\,e_- & 0 & 0 \\ \sqrt{2}\,e_+ & 0 & 0 & 0 \end{bmatrix} B Q - \begin{bmatrix} 0 & 0 & 0 & \sqrt{2}\,e_- \\ 0 & 0 & \sqrt{2}\,e_+ & 0 \\ 0 & \sqrt{2}\,e_- & 0 & 0 \\ \sqrt{2}\,e_+ & 0 & 0 & 0 \end{bmatrix} B_\nu \left. \right\} u_i$$

where $A = (e + q^2 F_1'(0) + \ldots)$; $B = (\mu_M + q^2 F_2'(0) + \ldots)$ and $e_\pm = (1/\sqrt{2})\ (e_1 \pm i e_2)$ are the polarization vectors.

For $q^2 = 0$ the electric charge term e in the electric form factor expansion is still present while all the others vanish. Since the symmetry is defined at $q^2 = 0$, this term cannot represent any symmetry breaking effect and must therefore be excluded from the analysis (1).

As a starting point we may choose a value for q^2 of the order of p^2 ($q^2 = -p^2$, since $q^2 < 0$). Then, using the solutions of the free particle Dirac equation for $\bar{u}$ and u and equations (2.5), (2.9), we have for the electric term in (3.4)

zero order

$$-\sqrt{2}\ e\ (-q^2)^{1/2} / 2M = -2.4 \times 10^{-7} \tag{3.5}$$

first order

$$-\sqrt{2}\ (-q^2)^{1/2} / 2M\ .\ q^2 F_1'(0) = 3 \times 10^{-8} \tag{3.5a}$$

which does not produce any symmetry breaking, as we realize upon comparison with p^2 (3.3).

---

(1) In fact one easily sees that it gives zero.

Turning to the magnetic terms in (3.4), we have similarly, using (2.5), (2.8), (2.10).

zero order

$$\sqrt{2}\ (q^2/2M)\ \mu_M\ (-q^2)^{1/2}/2M \simeq -8.7 \times 10^{-16} \qquad (3.6)$$

first order

$$\sqrt{2}\ (q^2/2M)\ (-q^2)^{1/2}/2M \cdot q^2 F_2'(0) \simeq 1.08 \times 10^{-16} \qquad (3.6a)$$

for the ν terms in (3.4) and

zero order

$$-\sqrt{2}\ [1 - q^2/4M^2]^{1/2}\ (-q^2)^{1/2}\ (4M^2 - q^2)^{1/2}\ \mu_M/2M \simeq -2.2 \times 10^{-4} \qquad (3.7)$$

first order

$$-\sqrt{2}\ [1 - q^2/4M^2]^{1/2}\ (-q^2)^{1/2}\ (4M^2 - q^2)^{1/2}\ q^2 F_2'(0)/2M \simeq -2.8 \times 10^{-5} \qquad (3.7a)$$

for the Q terms in (3.4).

Symmetry breaking is apparent here when we compare (3.7) with p^2 (3.3). Convergence of the series implies smaller values for higher order terms, therefore making it unnecessary to calculate further form factor derivatives.

Therefore, for transverse photons the symmetry breaking effect occurs overwhelmingly in the magnetic part of the amplitude. The smallest photon four-momentum able to produce it is easily determined from (3.7) being of the order of $p^2/16$. At this order of magnitude the contribution of all the other terms from the electric and magnetic part is negligible. In fact, substituting $q^2 = -p^2/16$ in (3.7), we have:

$$\sqrt{2}\ q^2 \mu_M = -5.5 \times 10^{-5} \qquad (3.8)$$

which is just over p^2 (3.3).

These results are summarized in Table I.

3.2 — *Symmetry breaking for longitudinal and time-like photons*

The interaction Hamiltonian is in this case:

$$\overline{S_3\, u_f} \left\{ \begin{bmatrix} 0 & 0 & (1+\nu/Q) & 0 \\ 0 & 0 & 0 & -(1+\nu/Q) \\ (1-\nu/Q) & 0 & 0 & 0 \\ 0 & (\nu/Q-1) & 0 & 0 \end{bmatrix} e_0 + \begin{bmatrix} -q^2/Q & 0 & 0 & 0 \\ 0 & -q^2/Q & 0 & 0 \\ 0 & 0 & q^2/Q & 0 \\ 0 & 0 & 0 & q^2/Q \end{bmatrix} e_0 \right\} u_i \qquad (3.9)$$

Whereas in the previous case the Lorentz condition $q_\mu\, e_\mu = 0$ did not provide any information, here it is essential for expressing e_0, e_3 in terms of kinematical quantities.

Expanding (3.9) and using equations (2.5) for ν and Q we obtain

$$\left[\left(1 - \frac{q^2}{4M^2} \right)^{1/2} - \frac{q^2}{2M\,(4M^2 - q^2)^{1/2}} \right] \left(e + \frac{q^2\, a^2}{8}\, e \right) + \frac{q^2}{(4M^2 - q^2)^{1/2}} \left(\frac{e}{4} + q^2\, \frac{a^3\, \mu}{16\,(\mu^2+1)}\, e \right) \qquad (3.10)$$

Proceeding now with the symmetry breaking analysis, we first note that, as regards the electric form factor, the first term cannot be accounted for, since it produces its largest contribution at $q^2 = 0$ and therefore does not represent a measure of the symmetry breaking effect for small q^2. Starting by choosing $q^2 = p^2$ as before, we have, for the electric part of the amplitude, using (2.9),

zero order

$$-\, e\, (q^2 / 2M)\, (4M^2 - q^2)^{-1/2} \simeq 3.36 \times 10^{-13} \qquad (3.11)$$

first order

$$-\, (q^2 / 2M)\, (4M^2 - q^2)^{-1/2}\, q^2\, F_1'(0) \simeq -\, 4.2 \times 10^{-14} \qquad (3.11a)$$

Comparing these values with p^2 (3.3) there is obviously no symmetry breaking in this case.

For the magnetic part, using (2.8), (2.10):

zero order

$$q^2 (4M^2 - q^2)^{-1/2} . (e/4) \simeq -3 \times 10^{-10} \qquad (3.12)$$

first order

$$q^2 (4M^2 - q^2)^{-1/2} . q^2 F_2'(0) \simeq 3.9 \times 10^{-11} \qquad (3.12a)$$

which does not give rise to any symmetry breaking as well. The smallest q^2 which is able to produce it, is, from (3.12a), of the order of 7p since in this case

$$(3.12a) \rightarrow 3.5 \times 10^{-5}$$

which is of the order of p^2 (3.3). The contribution from the other terms is again negligible. Therefore, for longitudinal and time-like photons, the symmetry breaking is much less drastic than for transverse ones and also determined by the magnetic part of the amplitude.

These results are summarized in Table II.

3.3 — *Discussion*

As we have seen the minimum photon momentum which is responsible for symmetry breaking is about 1.5×10^4 times greater for longitudinal and time-like photons than for transverse ones. It is worth noting that since, in the usual natural units system ($\hbar = c = 1$) the electron mass is $m = 0.511$ MeV, its Compton wavelength will be $\hbar / mc = (0{,}511 \text{ MeV})^{-1}$ resulting $p = 1/a = 3.72$ keV. Therefore the limit for transverse photons for instance is: $q^2 \sim p^2/16 = 0.86$ keV^2.

Equations (3.5) to (3.7a) and (3.11) to (3.12a) give us the information we require on the symmetry breaking. Higher than linear terms in q^2 are negligible provided this is kept small. The

TABLE I — For transverse photons breaking is determined by the magnetic part of the amplitude and always thereby dominated ($p = 1/a$).

	F_1				F_2					
$q^2 \rightarrow$	0	p^2	Limit p	p^0	0	p^3	Limit $p^2/16$	p^2	p	p^0
p^{-1}										B
p^0										
p				B					B	
p^2			B				B	B		
p^3	[B]	B			[B]	B				

results are summarized in Tables I, II where horizontally we plot q^2 values and vertically the order of magnitude of the corresponding terms in the Hamiltonian. The letter B denotes breaking at the corresponding order of magnitude. Thus BR symmetry breaking occurs everywhere above the row for p^3.

TABLE II — For longitudinal and time-like photons breaking is likewise determined and dominated by the magnetic part of the amplitude but much less drastic.

	F_1					F_2				
$q^2 \rightarrow$	0	p^2	p	p^0	Limit $2p^2$	0	p^2	p	Limit 7 p	p^0
p										B
p^2					B				B	
p^3	[B]	B	B	B		[B]	B	B		

One important remark should be made here concerning the $q^2 = 0$ limit: it is an assumption that at $q^2 = 0$ there is a residual symmetry breaking which is precisely BR's one (see equation (3.2)) and which is therefore always present. This residual breaking is denoted by the boxes enclosing the letter B in Tables I, II. It cannot be given by our model (which gives zero at $q^2 = 0$) and must therefore be taken as a starting point.

Finally, if one shifts the analysis to the final state rest frame by reversing the sign of ω in the boost matrix S_3 and boosting the initial state instead, it is easily seen that the results are quite insensitive to the shift for all the orders of q^2 considered in the tables.

3.4 — *Summary and conclusion*

Bell and Ruegg have defined an approximate symmetry which is broken by third order small terms at $q^2 = 0$ momentum transfer. This can be considered as a sort of residual breaking, always present, and denoted by the squares in Tables I, II. We investigated the effects of introducing $q^2 \neq 0$ carried by photons on the breaking of this approximate symmetry. The photon 4-momentum was compared to the internal momentum p of the constituents of the system (the hydrogen atom was taken as the working model). It appears that breaking is more drastic for transverse photons than for longitudinal and time-like ones. Thus, whereas these produce a limit $q^2 \sim 7\,p$, the limit for the transverse ones is $q^2 \sim p^2/16$, about 1.5×10^4 smaller.

These results are of course applicable to hadron-hadron scattering in the small q^2 sector. Here the quark-quark interaction can obviously not be treated in the one gluon exchange approximation and some further work is required so as to apply it in a realistic way to the confining-like sector of the interaction.

I am grateful to Dr. Hugh Burkhardt for his suggestion of this problem and help throughout the course of the work. I also wish to express my thanks to Dr. Jack Gunson for many valuable discussions and suggestions. Finally I am grateful to Instituto Nacional de Investigação Científica and Universidade Técnica de Lisboa for having provided financial support.

REFERENCES

[1] BELL, J. S. and H. RUEGG, *Nucl. Phys.* **B93** (1975), 12.

[2] DRELL, S. D. and F. ZACHARIASEN, «*Electromagnetic Structure of Nucleons*», Oxford University Press, 1961.

[3] LURIÉ, D., «*Particles and Fields*», John Wiley, 1968.

[4] ABERS, E. S., H. BURKHARDT, V. L. TEPLITZ, C. WILKIN, *Nuovo Cimento*, **42** (1966), 305.

[5] PULIDO, J., Ph. D. Thesis, University of Birmingham, 1978.

[6] FLUGGE, S., «*Practical Quantum Mechanics*», Springer-Verlag, 1974.

[7] BJORKEN, J. D. and S. D. DRELL., «*Relativistic Quantum Mechanics*», McGraw Hill, 1964.