MODERN TRENDS IN RESEARCH ON WAVES IN FLUIDS, PART I: GENERATION AND SCATTERING BY TURBULENT AND INHOMOGENEOUS FLOWS

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ABSTRACT — We consider the effects of turbulence and inhomogeneities upon waves in fluids (§1), both on extensive scales and as localized phenomena: on an extensive scale, in connection with the spectral and directional broadening of sound by turbulent and irregular shear layers separating a jet from a medium at rest (§ 2); as localized phenomena, producing forces and stresses, of hydrodynamic and electromagnetic origin, which are responsible for the generation of waves in fluids occuring in nature (§ 3). These problems are relevant in a variety of situations (Figures 1 to 4) of physical and engineering interest.

1 — INTRODUCTION

In the first part of this essay on some of the current trends in research on waves in fluids, we consider the effects of non-uniform and unsteady flow, leaving for the second part the consideration of external force fields. Most fluids commonly occurring in nature (Lamb 1879, Prandtl & Tietjens 1934, Landau & Lifshitz 1953, Batchelor 1967) give rise to flows containing inhomogeneities and turbulence (Batchelor 1953, Townsend 1956) in a variety of scales, and these disturbances affect the properties of waves (Rayleigh 1877, Brekhovskikh 1960, Morse & Ingard 1968, Whitham 1974, Levine 1978, Lighthill 1978), both as regards

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generation by localized phenomena and propagation in extensive regions.

Extensive inhomogeneous regions occur as interfaces, generally irregular, separating two media, solid or fluid, at rest or in relative motion; in the case of jets, the convection of the interface with the atmosphere at rest, may produce shear stresses, and thus entrain a layer of turbulence. The sound from sources in the interior of the jet is scattered by the interface, and diffracted by the turbulence, changing its directivity and spectrum. The combination of attenuation and interference effects leads to a directional and spectral broadening of an initially monochromatic beam; thus the reception of waves from a certain direction at a given frequency gives no assurance that there is a source of that frequency or radiating in that direction, unless the effects of scattering and diffraction on the spectrum and directivity have been taken into account.

Localized inhomogeneities and strong turbulence are associated with force and stress concentrations in a flow. The vector character of forces, and tensor character of stresses, suggests that they act respectively as dipole and quadrupole sources of the waves which are often observed in the fluids occuring in nature. Monopole radiation is also possible, in two-phase flow, e.g., a liquid with bubbles, as consequence of the variations in volume of one of the phases. The wave fields radiated by these mono-, di- and quadrupoles may be modified by boundaries constraining the flow or reflecting bodies deflecting the stream. In all cases, the estimation of the energy in the radiation field gives a measure of the effectiveness and importance of the wave generation mechanism in question.

2 — SPECTRAL AND DIRECTIONAL BROADENING BY SHEAR LAYERS

The classical wave equation, satisfied by the velocity perturbation $\mathbf{v}$ of an adiabatic acoustic wave in a medium at rest is (D'Alembert 1747, Rayleigh 1877):

$$\left\{ \partial^2/\partial t^2 - c^2 \nabla^2 \right\} \quad \mathbf{v}\,(\mathbf{x}, t) = 0, \tag{1}$$

where c is the adiabatic sound speed, which is constant for a perfect gas in a mean state of isothermal equilibrium. For a medium moving with a constant velocity $\mathbf{U}$, equation (1) is only valid in a frame $\mathbf{y}$ moving with the fluid; and returning to coordinates at rest, $\mathbf{x} = \mathbf{y} - \mathbf{U}\,t$, the Laplacian is unchanged, $\partial^2/\partial y_i^2 = \partial^2/\partial x_i^2$, but the local time derivative $\partial/\partial t$ is replaced by the material derivative, $d/dt = \partial/\partial t + U_i\ \partial/\partial x_i$, including the effect of the fluid motion $U_i\,\partial/\partial x_i = \mathbf{U} \cdot \nabla$. We obtain the convected wave equation (Morse & Ingard 1968, Lighthill 1978)

$$\{ (\partial/\partial t + \mathbf{U} . \nabla)^2 - c^2\ \nabla^2 \}\ \ \mathbf{v}\,(\mathbf{x}, t) = o, \tag{2}$$

describing the propagation of sound in an uniform flow of arbitrary velocity (subsonic, sonic or supersonic), and leading by the usual spectral methods, to Doppler (1893) effect, of changing the frequency ω to $\omega/(1\text{-}M \cos\Theta)$, where $M \equiv U/c$ is the Mach number of the flow and Θ the angle of the flow velocity with the direction of propagation.

In order to derive the wave equation in a non-uniform and/or unsteady flow, for which the mean velocity depends on space and/or time $\mathbf{U}\,(\mathbf{x}, t)$, a different method is required, using as variable (Howe 1975, Campos 1978a) the stagnation enthalpy Q, defined by:

$$dQ = d\,(\rho V^2/2) + \mathbf{E} \cdot d\mathbf{D} + \mathbf{B} \cdot d\mathbf{H} + \nu\, dN + (1/\rho)\ dp + T\ dS, \tag{3}$$

as the sum of the kinetic, electric, magnetic and chemical energies (per unit volume) with the Legendre transform $(1/\rho)$ dp of the mechanical work $-\,pd\,(1/\rho)$ and heat T dS; ρ denotes the mass density, $\mathbf{V} = \mathbf{U} + \mathbf{v}$ the total velocity of flow plus wave, $\mathbf{E}$, $\mathbf{D}$ the electric field and displacement, $\mathbf{B}$, $\mathbf{H}$ the magnetic induction and field, ν the chemical potential and N the mole number (for each chemical species), p the pressure and $1/\rho$ the specific volume, T the temperature and S the entropy.

In an homogeneous fluid of density ρ_0, not subject to external force fields, and in uniform motion at Mach number M, the stagnation enthalpy $Q \sim \{ (1 + M \cos\Theta)/\rho \}$ P scales on the acoustic pressure P, and thus is an acoustic variable. The exact wave equation statisfied by the stagnation enthalpy Q, in the absence of sources (which will be considered in § 3), in a variable flow of velocity $\mathbf{V}$ and pressure p, is:

$$\{ c^2 \, d/dt \, c^{-2} \, d/dt - \rho^{-1} \nabla p \cdot \nabla - c^2 \nabla^2 \} \, Q(\mathbf{x}, t) = 0, \quad (4)$$

where $d/dt \equiv \partial/\partial t + \mathbf{V} \cdot \nabla$ is the material derivative, and the sound speed c generally depends on position for a high-speed (or compressible) mean flow.

Considering a non-uniform but low Mach number $M^2 << 1$ (or incompressible) mean flow, so that the only compression of the medium is the acoustic wave, equation (4) is simplified since: (i) the sound speed $c = c_0 \{ 1 - (\gamma - 1) M^2/2 \}$, where γ is the ratio of specific heats (Landau & Lifshitz 1953), becomes constant and equal to the value in a medium at rest $c \sim c_0$; (ii) the pressure is approximated by the dynamic pressure $p \sim \rho V^2/2$, so that the second term of (4) is negligible compared with the third since $(\nabla p \cdot \nabla)/(\rho \, c^2 \, \nabla^2) \sim p/\rho \, c^2 \sim V^2/c^2 = M^2 << 1$. Neglecting the second term of (4), and setting c constant, it reduces to the convected wave equation, which thus also applies to non-uniform, low Mach number flows. In conclusion, the convected wave equation (2) describes the propagation of sound in: (i) an uniform flow of arbitrary velocity, including Mach number $M \sim 1$, in which case the mean flow, and not only the wave perturbation, is compressible; (ii) a non-uniform mean flow of low Mach number $M^2 << 1$, which is incompressible, leaving as sole compression the wave perturbation.

Thus the convected wave equation is adequate to describe the propagation, diffraction and scattering of sound in quite complex situations, e.g., the irregular and turbulent shear layer which separates a jet from a medium at rest, which can be modelled (Barratt, Davies & Fisher 1963) as: (i) an irregular interface moving at a fraction $\alpha \sim 0.6$ of the jet velocity $\mathbf{V}$, across which the density and sound speed change, and which scatters an incident wave into a reflected and transmitted wave, with reflection and transmission coefficients calculated from the continuity of pressure and displacement across the interface; (ii) the interface usually entrains turbulence, whose velocity consists of a constant global convection $\alpha \mathbf{V}$, and a local unsteady, non-uniform perturbation $\mathbf{u}(\mathbf{x}, t)$, whose r.m.s. value is typically a fraction $\beta \sim 0.15$ of the jet velocity, so that even for a bi-sonic jet $M \sim 2$, the turbulence is locally incompressible, $(\beta\, V/c)^2 \sim \beta^2 M^2 \sim (0.3)^2 << 1$, and (2) can be used to study the refraction of waves by the flow of velocity $\mathbf{U}(\mathbf{x}, t) = \alpha \mathbf{V} + \mathbf{u}(\mathbf{x}, t)$. The scattering of sound (or other waves) by irregular interfaces (Born & Wolf 1959, Sholnik 1962, Beckman & Spizzichino 1963, Berry 1973, Clarke 1973, Howe 1976, Campos 1978b) and the refraction in turbulence and other random media (Lighthill 1953, Philips 1960, Chernov 1967, Tatarski 1967, Howe 1973, Uscinski 1977, Ishimaru 1978, Campos 1978c), is relevant to the transmission of sound from the interior of jets (Mani 1976a, b, Balsa 1976, Jones 1977, Munt 1977, Howe & Ffowcs-Williams 1978, Campos 1984c), and thus to the problem of aircraft noise (Lighthill 1961, Nayfeh, Kaiser & Telionis 1975, Doak 1976, Crighton 1981, Campos 1984b) which is central to aerodynamic acoustics.

When sound is scattered by an irregular interface the phase of the reflected and transmitted wave depends on height of the scattering element, so that identical wave components scattered by different elements have different phases, i.e., a coherent incident beam is reflected and transmitted as an incoherent bundle. Random phase changes also occur when sound is refracted by turbulence, e.g., the aleatory magnitude and direction of the turbulent velocity causes random Doppler shifts of frequency, and an initially monaural beam (of a single frequency) is broadened into a widening spectrum. This phenomenon of spectral broadening of sound pulses propagating across turbulence and interfaces has been observed in air and water (Schmidt & Tillman 1970, Ho & Kovaznay 1967a, b, Beyer & Korman 1980, Juvé & Blanc-Benon 1981, 1982), and corresponds to the broadening of atomic emission lines (Rayleigh 1873, 1889, 1915) of the Bunsen burner, and can be modelled (Campos 1978b, c) by: (i) determining the acoustic pressure field emitted by a source in a jet and transmitted across the turbulent and irregular shear layer to the outside; (ii) calculating the energy radiated at each frequency and each direction, from the correlations of the acoustic pressure, which in turn depend on the statistical properties (Lindeberg 1922, Khinchin 1948, von Mises 1960) of the shear layer.

Fig. 1 shows the good agreement of the spectra calculated theoretically (Campos 1978b, c; Campos 1984c) using a computer program in Cambridge's IBM 370 and 8031 (large plot), with the

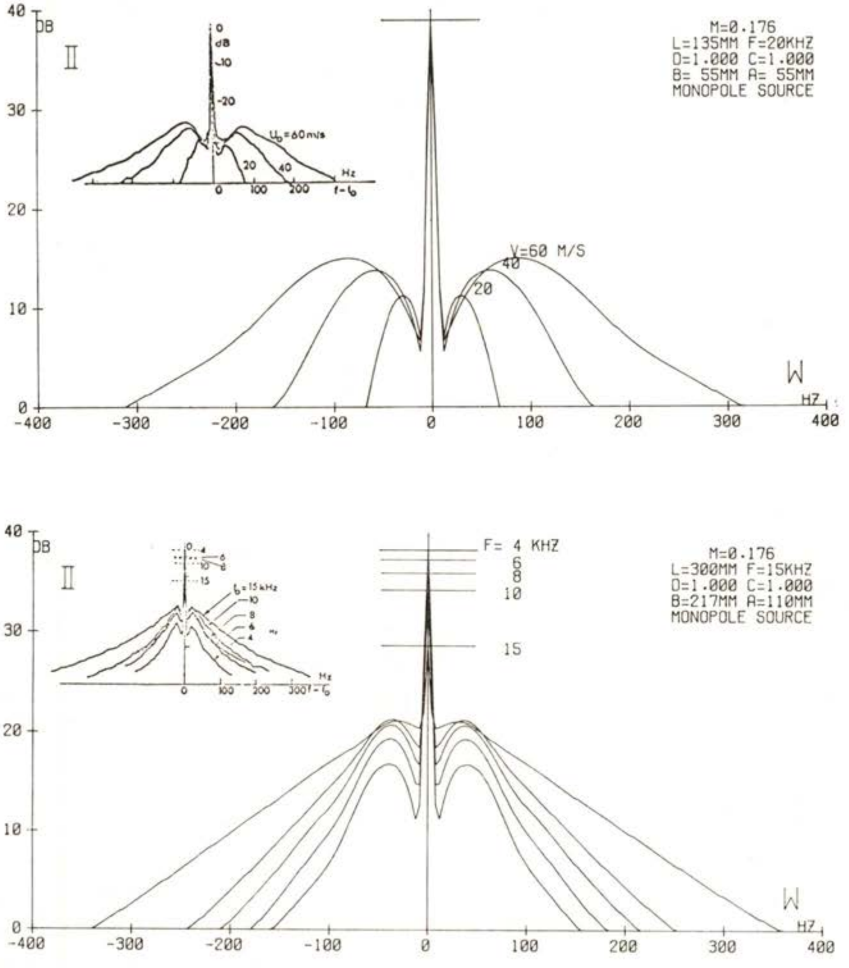
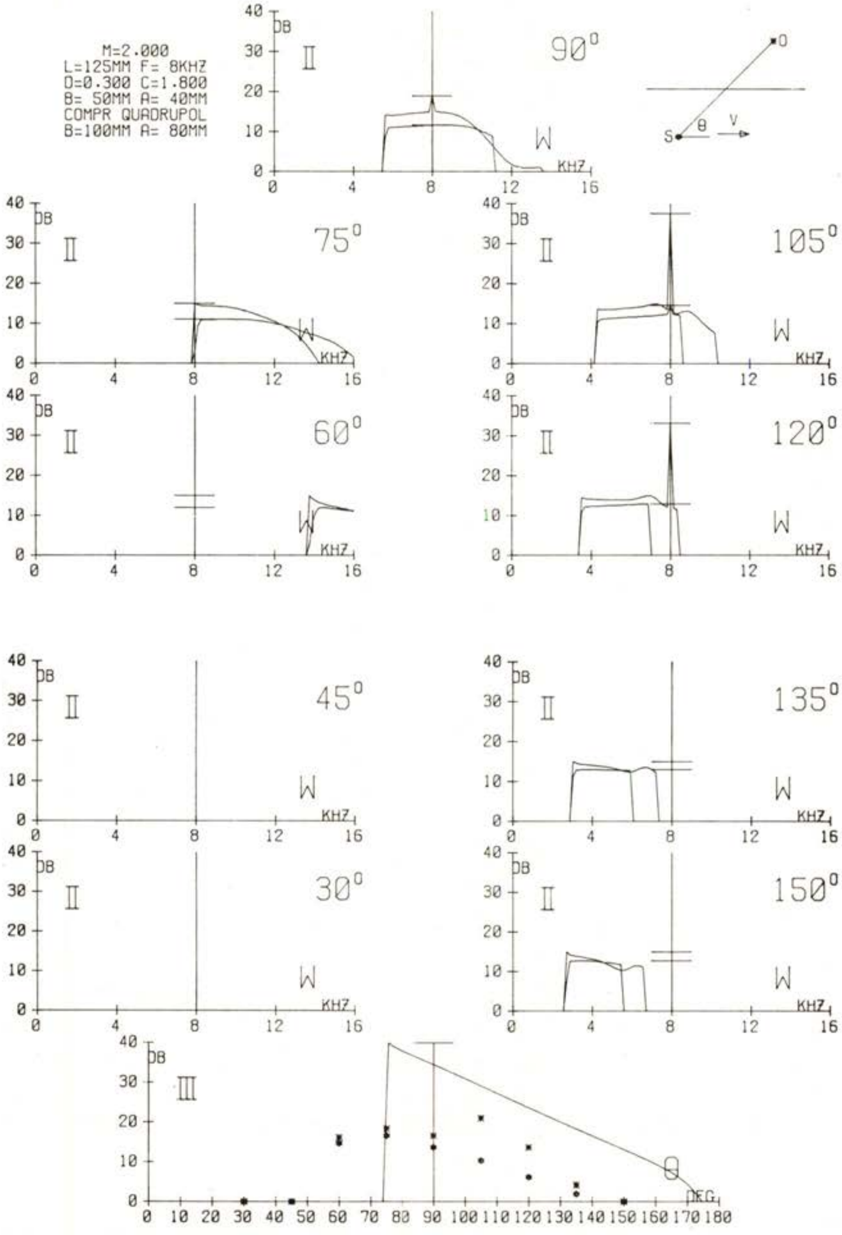


Fig. 1 — Spectra from a monochromatic point source in a cold, low-spped air jet, received outside, after transmission across the irregular and turbulent shear layer separating the jet from the ambient medium at rest. The experimental spectra (inset) were measured by Candel, Guédel & Julienne (1975) in an anechoic chamber at the von Karman Institute; the theoretical spectra (plot) were computed in the IBM 370-8031 in Cambridge University, using the theory of Campos (1978a, b; 1984c). The spectra show the effects of increasing the jet velocity (top) in broadening the spectra, and of increasing the source frequency (bottom) in attenuating the spike and reinforcing the broadband.

measurements (Candel, Julienne & Julliand 1975, Candel, Guédel & Julienne 1976) of sound sources in jets (smaller inset graph). Although the sources are monochromatic, with a frequency corresponding to that of the 'spike', the spectrum received after transmission across the turbulent and irregular shear layer shows the effects of spectral broadening in the appearance of prominent side bands, in a frequency range absent from the source. The effect of increasing the jet velocity, for the same source, is seen (top) in more intense and separated sidebands and a spectrum wider overall; if the frequency of the source is increased, for the same jet velocity, then (bottom) the spike is attenuated, and part of the energy transfered to the sidebands, that become wider and higher. The theory can be used to calculate the spectra received outside the jet in any direction, for a source with arbitrary spectrum, spatial distribution and multipolar character, within cold or hot jets, for various scales of turbulence and irregularity in the shear layer.

Having validated the theory by comparison with measurements on cold, low-speed air jets in anechoic chambers, we proceed to the prediction of the noise spectra of hot, high-speed jet exhausts, taking as an example Concorde in the take-off configuration. The cases of (i) a vertical dipole source representing sound emission by patches of unburned gas in the exhaust (Howe 1975, Campos 1978a), and (ii) a shear quadrupole modelling noise generation by turbulence in a jet (Lighthill 1952, 1954; Campos 1977), have been illustrated elsewhere (Campos 1978c, 1984c). Here we consider another turbulent source of sound, namely, a compressive quadrupole, of frequency 8 kHz corresponding to the turbine tone of the Olympus 593D turbojet of Concorde, and the corresponding "spectra and directivity data sheet" is given in Fig. 2. The spectra II are given in nine directions spaced 15° between 30° and 150°, for a shear layer (a) whose statistical properties roughly correspond to those of Concorde's exhaust, and for (b) a shear layer twice as irregular and thick: (i) the effects of spectral broadening are considerable, since the sound from the monochromatic source of frequency 8 kHz is received over the spectral band 3-16 kHz; (ii) for the single irregular and turbulent shear layer, there is a prominent 'spike' at source's frequency in the directions 105-120°, just *before* the aircraft flies by, this discrete tone being attenuated into the band


M=2.000
L=125MM F= 8KHZ
D=0.300 C=1.800
B= 50MM A= 40MM
COMPR QUADRUPOL
B=100MM A= 80MM
DB
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90°
75°
105°
60°
120°
45°
135°
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II
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in other directions; (iii) if the shear layer is made twice as irregular, and the region of entrained turbulence becomes twice as thick, then the acoustic energy in the spike is scattered into the bands in all directions, and the source's tone is just barely visible at 105°, where before it was most prominent.

The directivity III, or total energy received at all frequencies is plotted (Fig. 2, bottom) as a function of direction in three cases: (solid line) a plane vortex sheet between the same jet and environment, which radiates only a 'spike' at source's frequency (no broadband), and has a downstream 'zone of silence' $\theta < 74°$; (asterisks) the irregular and turbulent shear layer radiates a band into what would be the 'zone of silence' of the plane vortex sheet, and in all other directions radiates a band and 'spike', with the latter showing the importance of attenuation by the irregularities of the interface and the layer of entrained turbulence; (stars) the shear layer twice as thick and irregular gives slightly lower noise levels (~ 3 dB) in all directions except the range 105-120°, where the absorption of the formerly prominent spike into the band yields a significant attenuation ~ 10-12 dB. We conclude that: (i) the model of the plane vortex sheet between the jet and a medium at rest is at fault, since it cannot predict spectral broadening or radiation into the 'zone of silence', and overestimates peak noise levels by up to 20 dB as a consequence of neglecting attenuation mechanisms: (ii) the irregular and turbulent shear layer can reduce the audible noise disturbance, by distributing the acoustic energy over a wider range of directions, including the 'zone of silence', and scattering 'spikes' into spectral broadbands.

Fig. 2 — Simulation of the transmission of sound from a compressive quadrupole source, at the frequency 8 kHz of the turbine tone of the Olympus 593D turbojet, modelling turbulence in the jet exhaust of Concorde in the take-off configuration. The spectra II received outside are given in nine directions, for a single turbulent and irregular shear layer, and for a shear layer of double thickness which provides increased attenuation of spikes. The directivity III, or spectrum integrated over all frequencies, is plotted (at the bottom) as function of direction, for the single (asterisks) and double (stars) irregular and turbulent shear layers, in comparison with an idealized plane vortex sheet (solid line) between the same media.

These methods of attenuation of jet noise have been used in successive generations of transport aircraft: (I) the first jet airliners used multi-lobe nozzles, which produce an irregular shear layer, and entrain a layer of turbulence, to scatter and diffract sound; (II) the current wide-body airliners use high by-pass ratio turbofans, with the hot, high-speed core jet exhaust surrounded by a cold, low-speed by-pass flow, that acts as a double shear layer, providing greater sound attenuation; (III) in future jet-powered aircraft, the methods of noise reduction based on scattering by irregular interfaces and diffraction by turbulent layers have to be harmonized with reduced fuel consumption and increased propulsion efficiency.

3 — EMISSION BY HYDRODYNAMIC AND ELECTROMAGNETIC FORCES AND STRESSES

The classical theories of generation of acoustic waves by mass or momentum sources, e.g., sirens, pipes, membranes, or of emission of electromagnetic waves by electric charges and currents, assume that the sources of waves are physically separate from the medium of propagation, and they appear as an inhomogeneous, forcing term in the wave equation. The modern theories of wave generation in natural (atmosphere, ocean) or engineering (jets, exhausts) fluids, have to contend with a situation where the sources cannot be identified or isolated 'a priori', since they lie in the medium of propagation. In order to identify the physical processes whereby waves are generated, it is necessary to start from the general equations of fluid motion under force and stress fields, consider the flow as the superposition of a mean state and a wave perturbation, say, represented by the velocity $v_i(\mathbf{x}, t)$, and: (i) separate the linear terms, and eliminate between them for the velocity, thus obtaining an expression $\Box_{ij}\, v_j = o$, which could describe propagation in the absence of sources or damping, and thus allow us to identify the wave operator as $\Box_{ij}$; (ii) retain all the non-linear and dissipative terms, and group them as forcing the wave equation, viz.:

$$\begin{aligned} &\{ \Box_{ij}(\partial/\partial t, \partial/\partial x_i) \}\, v_j(\mathbf{x}, t) \\ &\qquad = -\rho^{-1}(\partial/\partial t)\{ \partial Q/\partial x_i + P_i + \partial R_{ij}/\partial x_j \}, \end{aligned} \tag{5}$$

where the terms on the right-hand-side can be interpreted as sources, respectively of monopole Q, dipole P_i and quadrupole R_{ij} character.

Considering waves under the combined influences of compressibility, magnetism and gravity, we have (Yu 1965, McLellan & Winterberg 1968, Bel & Mein 1971, Stein & Leibacher 1974, Campos 1977, 1983b, 1984a), the magneto-acoustic-gravity wave operator:

$$\Box_{ij} = \delta_{ij}\, \partial^2/\partial t^2 - c^2\ \partial^2/\partial x_i\, \partial x_j - g_j\ \partial/\partial x_i - (\gamma - 1)\ g_i\ \partial/\partial x_j$$
$$- a^2 (\delta_{ij}\partial^2/\partial m^2 - m_j \partial^2/\partial m\, \partial x_i) + a^2 (m_i \partial^2/\partial m\, \partial x_j - \partial^2/\partial x_i\, \partial x_j), \tag{6}$$

where c, a denote respectively the sound and Alfvèn speeds:

$$c^2 = \gamma\, RT, \qquad a^2 = \mu\ H^2/4\,\pi\,\rho\ , \tag{7a, b}$$

where γ is the ratio of specific heats, R the gas constant, T the temperature, μ the magnetic permeability, ρ the density, $\mathbf{g}$ the acceleration of gravity, and $\mathbf{H}$ the constant external magnetic field of direction $\mathbf{m} \equiv \mathbf{H}/H$, so that the derivative along magnetic field lines is $\partial/\partial m = m_i \partial/\partial x_i = \mathbf{m} \cdot \nabla$. The terms in (6) may be interpreted as follows, from left to right (Bray & Loughhead 1974, Athay 1976, Campos 1983a, b): (i) second order time dependence allowing wave propagation in opposite directions and their superposition into standing modes; (ii) acoustic propagation, involving the sound speed c and fluid dilatation $\partial v_j/\partial\ _j = \nabla \cdot \mathbf{v}$ (iii) internal wave propagation, involving the acceleration of gravity g_j, (iv) acoustic-gravity coupling, through the gravity g_j and dilatation; (v-vi) magnetic wave propagation along magnetic field lines $\partial/\partial m$ at Alfvèn speed a; (vii-viii) magneto-acoustic coupling through the Alfvèn speed a and dilatation $\partial v_j/\partial x_j$, the former involving gravity effects through the density ρ stratification.

The theories of aerodynamic generation of sound (Lighthill 1952, 1954; Curle 1955; Powell 1968; Ffowcs-Williams & Hawkins 1968; Crighton & Ffowcs-Williams 1969; Howe 1975; Dowling, Ffowcs-Williams & Goldstein 1978; Campos 1978a), have been extended to stratified (Parker 1964, Stein 1967, 1981) and to ionized

(Kulsrud 1955, Lighthill 1967, Campos 1977) fluids, in connection with acoustic-gravity and magneto-acoustic waves. Considering the generation of the latter we have the following dynamic and magnetic quadrupole sources: (i) the anisotropic Reynolds and Maxwell stresses minus the isotropic dynamic and magnetic pressures:

$$R^{(1)}_{ij} = \rho\, v_i\, v_j - \rho\, v^2/2\ \delta_{ij}\,, \tag{8a}$$

$$S^{(1)}_{ij} = -(\mu/4\pi)\ h_i\ h_j + (\mu h^2/8\pi)\ \delta_{ij}\,, \tag{8b}$$

which scale quadratically on the velocity $\mathbf{v}$ and magnetic field $\mathbf{h}$ perturbations, and model the generation of waves by hydromagnetic turbulence; (ii) the inhomogeneous terms in the equation of state and Alfvèn equation:

$$R^{(2)}_{ij} = \{\, p - (\partial p/\partial \rho)_s\ \rho - (\partial p/\partial s)_\rho\ s \,\}\ \delta_{ij}\,, \tag{9a}$$

$$\begin{aligned} \partial S^{(2)}_{ij}/\partial t = -(\mu/4\pi)\ \{\, & H_j\ \nabla\wedge(\mathbf{v}\wedge\mathbf{h})_i + H_i\ \nabla\wedge(\mathbf{v}\wedge\mathbf{h})_j \\ & - \mathbf{H}\cdot\nabla\wedge(\mathbf{v}\wedge\mathbf{h})\ \delta_{ij}\,\}\,, \end{aligned} \tag{9b}$$

which would vanish (9a) for homogeneous, isentropic acoustics since then there is a single equation of state $p(\rho, s)$, and (9b) for Alfvèn waves in a perfectly conducting medium since then $\mathbf{v}\wedge\mathbf{h} = 0$, so that these terms represent the emission of waves by ionized inhomogeneities.

The dissipation tensor has terms corresponding to the viscous stresses τ_{ij}, thermal conductivity κ and electrical resistivity $1/\sigma$ (where σ is the Ohmic conductivity):

$$\begin{aligned} \partial D_{ij}/\partial t = & \{\, \partial\tau_{ij}/\partial t - \tau_{kl}\ (\partial v_k/\partial x_l)\ \beta\ \delta_{ij}\,\} - \kappa\ \nabla^2 T\ \beta\ \delta_{ij} \\ & + (c_*^2/16\,\pi^2\,\sigma)\ \{\, H_i\ \nabla^2 h_j + H_j\ \nabla^2 h_i - (\nabla\wedge\mathbf{h})^2\ \beta\ \delta_{ij}\,\}\,; \end{aligned} \tag{10}$$

where c_* is the speed of light in vacuo and β is the thermodynamic parameter $\beta \equiv (\rho T)^{-1}\ (\partial p/\partial s)_\rho$, which is a constant $\beta = 2/N$ for a perfect gas whose molecules have N degrees of freedom, i.e. $\beta = (2/3,\ 2/5,\ 1/3)$ for mono-, di- and polyatomic gases. The dissipation tensor (10) causes damping of the waves in the propagation region, and diffuses the sources in the generation region.

The hydrodynamic (8a, 9a) and magnetic (8b, 9b) sources scale respectively on the dynamic $R \sim \rho\, v^2/2$ and magnetic $S \sim \mu h^2/8\pi$ pressures, and comparing with the viscous D_ν, thermal D_K and resistive D_σ terms in the dissipation tensor (10), we obtain:

$$\frac{D_\nu/R}{1-\beta} \sim \frac{\nu}{\rho\, U\, L} \equiv \frac{1}{Re}\,, \frac{D_\kappa/R}{-\beta} \sim \frac{\kappa/C_p}{\rho\, U\, L} \equiv \frac{1}{Pe}\,, \frac{D_\sigma/s}{1-\beta} \sim \frac{c_*^2/4\pi\mu}{\sigma\, U\, L} \equiv \frac{1}{Me}\,, \tag{11a, b, c}$$

where Re, Pe, Me define respectively the Reynolds, Péclet and magnetic Reynolds numbers. If Re, Pe, Me < 1 the dissipation predominates over the generation of waves, otherwise it diffuses some of the energy flux of the sources. The momentum equation $\partial R_{ij}/\partial_{\ j}$ shows that if the forces P_i integrated over the wave generation region do not vanish, the source is equivalent to a dipole P_i ; an energy source Q would correspond to a monopole, and all three appear in the forcing term of the complete wave equation (5).

Provided that we consider linear waves in an homogeneous medium, even if the modes are anisotropic, dispersive or dissipative, and the source is any combination of multipoles, equation (5) can be solved by Fourier analysis and the asymptotic solution found explicitly using the method of stationary phase (Lighthill 1960, 1964, 1978; Ffowcs-Williams & Hawkins 1968; Adam 1982; Campos 1977, 1983d). The solution distinguishes non-dissipative waves, and cases where the wavenumber surface $k_1(k_2, k_3, \omega)$ is flat, has a single or a double curvature, i.e., is plane, cylindrical or spheroidal, the appearance of inflexion edges or caustics being also amenable to the method. We give the example of a three-dimensional, non-dissipative wave due to a quadrupole source, for which the asymptotic velocity perturbation is given by:

$$v_i(\mathbf{x}, t) \sim \Sigma\,(4\,\pi^2\, i/r)\ |\,g\,|^{-1/2} \left\{ \omega\ k_j\ \hat{T}_{jk}\ \Lambda_{ki}/\,|\,\partial\Lambda/\partial\mathbf{k}\,|\right\} \exp\left\{ i\,(\mathbf{k}\,.\,\mathbf{x} + \Phi)\right\}, \tag{12}$$

where: (i) the summation extends to all wavemodes and to all points on the wavenumber surface where the group velocity $\partial\omega/\partial\mathbf{k}$ points to the observer; (ii) $\omega, \mathbf{k}$ are the frequency and wavevector, $\hat{T}_{ij}$ is the spectrum of the source quadrupole, Λ_{jk} the inverse of the dispersion matrix, and $\partial\Lambda/\partial\mathbf{k}$ the derivative of its determinant in wavenormal direction; (iii) the asymptotic field decays with distance r like r^{-1}, and radiation occurs over a fraction of the solid angle 4π determined by the inverse square root of the Gaussian curvature g ; (iv) the emission phase $\mathbf{k}\,.\,\mathbf{x}$ is unchanged by the focal phase $\Phi = o$ for anticlastic beams whose principal curvatures are of opposite signs $g = g_1\, g_2 < o$, but for synclastic beams $g > o$ the focal phase is $\Phi = \pi/2$ in the divergent $g_1 , g_2 > o$ and $\Phi = -\pi/2$ in the convergent $g_1 , g_2 < o$ case, so that passage through a focus causes a phase jump $\Delta\,\Phi = \pi$.

In the case of dipolar sources, the stress quadrupoles (8, 9a, b), can be replaced by forces: the electromagnetic dipole is the Laplace-Lorentz force $\mathbf{F}$, whereas the hydrodynamic dipole is the hydrodynamic force $\mathbf{G}$:

$$\mathbf{F} = q\,\mathbf{E} + (\,1/c_*\,)\ \mathbf{J}\wedge\mathbf{B}\ ,\ \mathbf{G} = [\,(\,\rho/\rho_0\,) - 1\,]\,\nabla p + (\,\nabla\wedge\mathbf{v}\,)\wedge\rho\mathbf{v}\ , \tag{13a, b}$$

and considering an ionized inhomogeneity there is an analogy between: (I) the inhomogeneous force equal to the dimensionless $\delta \equiv \rho/\rho_0 - 1$ density difference between fluid ρ and blob ρ_0 times the pressure gradient ∇p , and the electric force on a charge q due to the electric field $\mathbf{E}$; note that in the electrostatic case the electric field $\mathbf{E} = \nabla\Phi$ is the gradient of a potential Φ , and takes the role of the pressure gradient ∇p , and a positive $q > 0$ / negative $q < 0$ charge corresponds $\delta > / < 0$ to a blob respectively lighter $\rho_0 < \rho$ / denser $\rho_0 > \rho$ than the fluid, which is compressed by/expands against the flow; (II) the Lamb's force on vorticity $\mathbf{w} = \nabla\wedge\mathbf{v}$ crossing streamlines $\mathbf{w}\wedge\mathbf{v}$, and the magnetic force on currents $\mathbf{J}$ transverse to the induction field lines $\mathbf{B}$; in the magnetostatic case the electric current is related to the curl of the magnetic induction $(\,1/c_*\,)\ \mathbf{J} = (\,\mu/4\pi\,)\ \nabla\wedge\mathbf{B}$, much as the vorticity to the velocity $\mathbf{w} = \nabla\wedge\mathbf{v}$, and the mass density ρ is replaced by $\mu/4\pi$, where μ is the magnetic permeability, which plays the role of a 'magnetic mass'. This can be seen by noting that if the mass density ρ is replaced by the magnetic permeability $\mu/4\pi$, and the velocity $\mathbf{V}$ by the magnetic field $\mathbf{H}$, the dynamic pressure $p_v = \rho V^2/2$ corresponds to the magnetic pressure

$p_h = (\mu/8\pi)\ H^2$ and the Reynolds stresses (8a) correspond to (8b), the Maxwell stresses. Thus we have extended the classical analogy between electro- and magnetostatic fields (Jeans 1908, Stratton 1944, Jones 1964) and potential flow (Lamb 1879, Landau & Lifshitz 1953, Milne-Thomson 1958) from the field variables and sources to the hydrodynamic and electromagnetic forces (13a, b) and stresses (8a, b).

As the ionized inhomogeneity is convected by the flow past the external fields, it describes a trajectory $\mathbf{y}(\tau)$ with velocity $\mathbf{U}$, and is acted upon by hydrodynamic (13b) and electromagnetic (13a) forces, that perform an activity (work per unit time) given by $(\mathbf{G}/\rho + \mathbf{F}/\rho_0)\cdot\mathbf{w}$, where $\mathbf{w}$ is the group velocity. If the activity is conserved along the trajectory, there is no net exchange of energy between the blob and the surrounding fluid, no sound is emitted, and the blob is convected silently. Conversely, if the activity varies along the blob's trajectory, then the excess or default of energy, in the absence of dissipation, is transferred to the surrounding fluid by expansion or contraction, i.e., the blob emits a sound pulse whose acoustic pressure is given asymptotically, for an observer in the far field, by:

$$\begin{aligned} P(\mathbf{x}, t) &\sim \{ 4\pi c^2\ |\mathbf{x}|\ (1 + M\ \cos\Theta) \}^{-1} \\ &\{ \partial/\partial\tau + \mathbf{U}\cdot\partial/\partial\mathbf{y} \}\ \{ (\mathbf{F}/\rho_0 + \mathbf{G}/\rho)\cdot\mathbf{w} \}, \end{aligned} \tag{14}$$

where $(\mathbf{y}, \tau)$ are the position and time of emission, and $(\mathbf{x}, t)$ those of reception. In order to determine the shape of the sound pulse it is necessary to calculate in detail the sound field (14), for example, for an ionized inhomogeneity (blob) which is: (i) convected along the streamlines of a flow, e.g., potential, past an electrified body, say, a sphere with surface electric currents; (ii) the presence of the body deforms the incident flow, and creates a pressure gradient, which exerts on the blob (13b) a displacement force $(\rho/\rho_0 - 1)\nabla p$, which acts as an hydrodynamic dipole source; (iii) the electric currents on the body create an external magnetic field, which exerts (13a) a Lorentz force $(1/c_*)\ \mathbf{J}\wedge\mathbf{B}$ on the blob, and acts as an electromagnetic dipole source; (iv) the total, hydrodynamic plus electromagnetic, dipole source radiates directly to the observer in the far-field, and is modified by the wave reflected from the body, which is also dipolar, and hence of comparable magnitude.

In Fig. 3 are illustrated the hydrodynamic sound pulses emitted by the blob streaming past the neutral sphere, received at two positions in the far-field: (left) for an observer midstream (at $\Theta = 90°$, above the sphere), the pulse is symmetrical, since both the direct and reflected wave are; (right) for an observer downstream (at $\Theta = 180°$) the pulse has lost its symmetry as a consequence of the asymmetry of the reflected wave. In Fig. 4 we show the electromagnetic sound pulses received, in the far-field in the midstream ($\Theta = 90°$) direction, due to electric currents on the sphere, whose axis is either alligned with (left) or perpen-

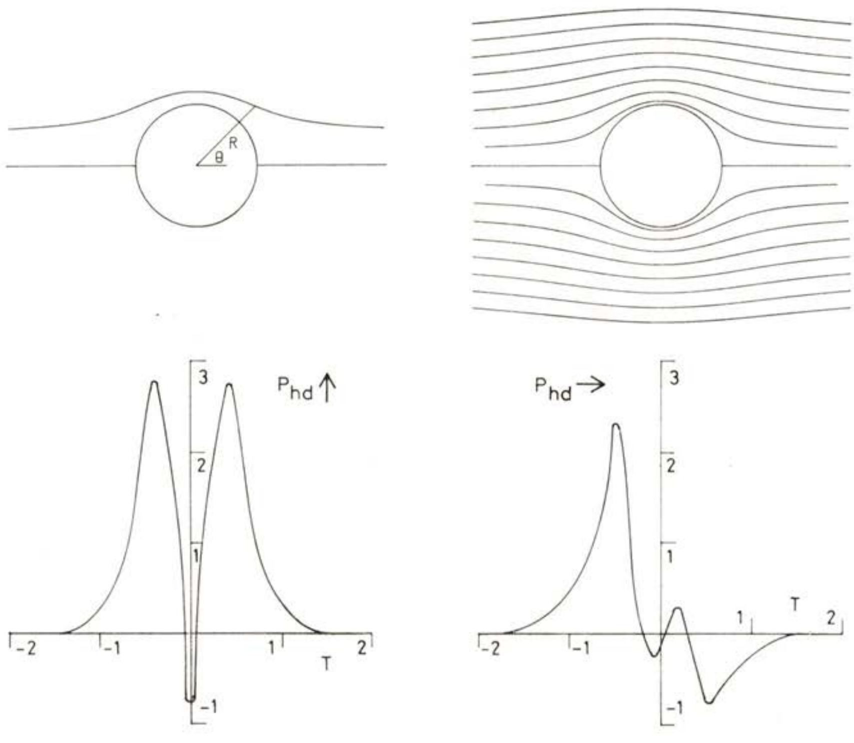

Fig. 3 — Emission of hydrodynamic sound by an inhomogeneity (blob) convected by a potential flow past a sphere, along the streamline of aiming distance half the sphere's radius. The sound pulse is plotted as received by an observer in the far-field in two directions: (left) midstream ($\Theta = 90°$), i.e., along the perpendicular to the incident flow direction, passing through the centre of the sphere; (right) downstream ($\Theta = 180°$), i.e., along the axis of the flow.

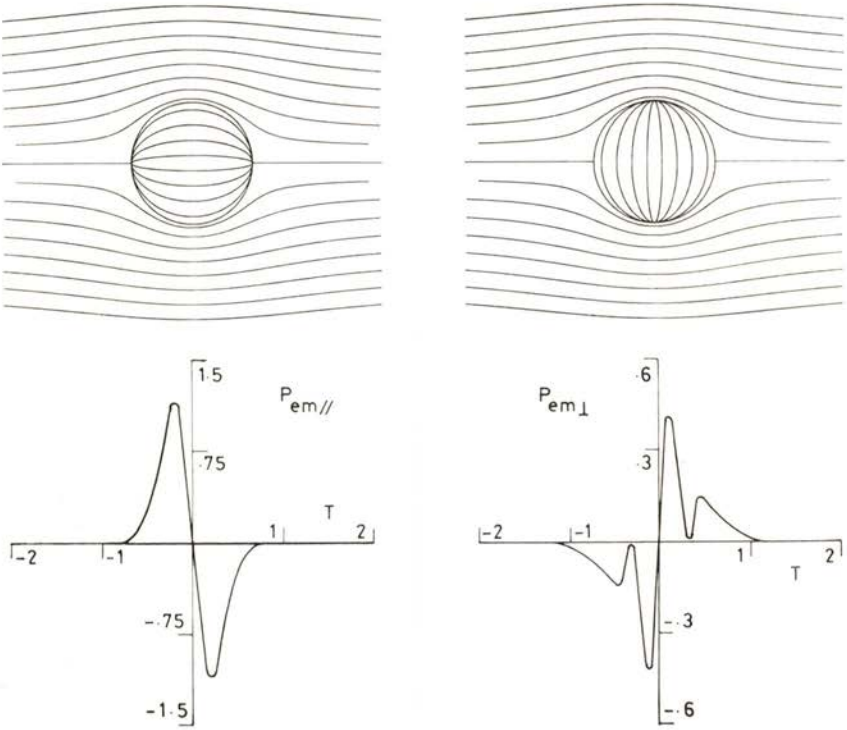

Fig. 4 — Emission of electromagnetic sound by an ionized blob as it is convected by the flow through a non-uniform magnetic field created by electric currents on the sphere. The sound pulse is plotted as received by an observer in the far-field in the midstream direction, for the cases in which the axis of the electric currents is parallel (left) and perpendicular (right) to the direction of the incident flow.

dicular to (right) the flow; the sound pulses are anti-symmetrical, due to the skew-symmetry of the magnetic field and force. As an example of interpretation of a pulse, we consider the hydrodynamic sound received midstream (fig. 3, left): (i) as the blob approaches the sphere it encounters first a region of diverging streamlines, hence reducing velocity and increasing pressure, so that the positive pressure gradient causes the emission of a compression pulse; (ii) as the blob is convected nearer to the sphere the streamlines converge, the velocity increases and the pressure reduces, and the reversal of the pressure gradient is signalled by emitting a rarefaction pulse; (iii) as the blob goes past the point of closest approach to the sphere the process is reversed both as concerns the direct and reflected wave, and a pulse symmetric overall results.

From the wave fields (12) or (14) the total intensity of radiation can be calculated, and in the general case when the gas $\rho U^2/2$ and magnetic $\mu H^2/8\pi$ pressures are comparable, it scales as:

$$I \sim (L^2/\rho\overline{c})\ (M^0, M^2, M^4)\ (\rho U^2 + \mu H^2/4\pi)^2, \qquad (15)$$

respectively for mono-, di- and quadrupole sources, where L is the linear size of the source region, $\overline{c}$ the phase speed and $M \equiv U/\overline{c}$ the Mach number measuring the relative amplitude of the wave. If the magnetic field is absent or the magnetic pressure negligible we obtain the acoustic intensity I_v scaling on the fourth power of velocity (16a), whereas if the magnetic pressure predominates (e.g., in a rarefied ionized fluid under strong magnetic fields), we obtain the hydromagnetic intensity I_h, which scales on the fourth power of the magnetic field:

$$I_v \sim (\rho L^2/\overline{c})\ U^4\ (M^0, M^2, M^4), \qquad (16a)$$

$$I_h \sim (\mu^2 L^2/\rho\overline{c})\ H^4\ (M^0, M^2, M^4), \qquad (16b)$$

with factors 1, M^2, M^4 respectively for mono-, di- and quadrupoles. The scaling of the noise of turbulent jets of low Mach number $M^2 << 1$ on the eighth power of the velocity (16a) $M^4 U^4 \sim U^8$ has been demonstrated experimentally (Lighthill 1954), confirming the predicted low efficiency (Lighthill 1952) of the quadrupole generation mechanism. Thus sound emission is dominated by the more efficient $\sim M^2$ dipole process $\sim M^2 U^4 \sim U^6$, in the presence of inhomogeneities convected by the mean flow (Howe 1975, Campos 1978a); even if the latter are absent, the presence of solid boundaries acting as dipolar reflectors can enhance the far-field of quadrupole turbulence sources (Curle 1955), e.g., a loose panel in an aerodynamic tunnel can be noisier than the flow itself. The greatest efficiency $\sim M^0$, and higher intensity $\sim U^4$, corresponds to monopole emission in two-phase flow (Crighton & Ffowcs-Williams 1969), of which cavitation noise is an example. Thus the dipolar effect of boundaries confining the flow and reflecting the sound is negligible for monopole sources, dominant for quadrupole sources, and of comparable magnitude to direct emission for the dipole source case.

The scaling laws for the intensity of radiation discussed above have an extensive field of application: (16a) applies to the aerodynamic acoustics of jets, and has been used in connection with the estimation of sound radiation by turbulence and inhomogeneities in free and confined flows; (16b) is of analogous form replacing the gas by the magnetic pressure, and applies to hydromagnetic waves in ionized fluids under dominant magnetic fields, e.g., to magnetohydrodynamic energy generators and thermonuclear fusion devices, where the plasma is confined by strong magnetic fields; (15) includes the case of comparable kinetic and magnetic pressures, and the transitions to dominance by one or the other, and applies in a variety of situations, from the molten interior mantle to the ionized upper layers of the atmosphere of the earth, and to phenomena in the sun and other stars, clusters, galaxies and matter scattered through the vast expanses of the universe.

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