SEMICONDUCTOR PROPERTIES OF THE EARTH'S CORE

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ABSTRACT-Assuming that the Earth's core consists of a nickel-iron oxide, which is certainly a semi-conductor, some relevant electrical properties of such material are presently discussed. Under convenient conditions, the actual geomagnetic field can be generated by a small rotation of the solid inner core in relation to the mantle.

1. INTRODUCTION

It is currently believed that the material in the Earth's core is a nickel-iron alloy (for e.g. $Fe_{0.9}Ni_{0.1}$,) with addition of an element of lower atomic weight. For some time, this additional element was supposed to be sulphur, but now oxygen is receiving considerable favour [1,2].

As iron oxides are semiconductors, the hypothesis gives the core remarkable electrical properties and suggests an adequate mechanism for generating the geomagnetic field [3]. This appears to be a promising alternative to the self-exciting dynamo which is currently accepted, but could not yet receive a quantitative treatment [4].

2. THERMOELECTRIC PROPERTIES OF SEMICONDUCTORS

In a semiconductor, electric current is due to the movement of both electrons and holes. With n electrons and p holes the flow of electric charge is

$$j_q = (n\mu_e + p\mu_h)eE \qquad (1)$$

where μ_e, μ_h are, respectively the mobilities of electrons and holes; e is the elementary charge and E is the electric field [5].

On the other hand, semiconductors are characterized by their energy bands (Fig. 1). Let E_c be the lowest point of the conduction band, and E_v the highest point of the valence band.

The energy gap is

$$E_g = E_c - E_v \qquad (2)$$

and the Fermi level ε_F lies in this gap so that we can write

$$\varepsilon_F = E_v + f\,E_g \qquad (3)$$

with $0 < f < 1$.

At absolute zero, the conduction band is vacant and the valence band completely filled. As temperature rises, some electrons from the valence band are excited and pass into the conduction band. Following Kittel [5], we assume that the electrons acquire an energy $E_c - \varepsilon_F$ and the holes, which are left behind in the valence band, acquire energy $\varepsilon_F - E_v$. In both cases we must add the thermal energy $\frac{3}{2} k_B T$, T being the temperature and k_B the Boltzmann constant.

The total flow of energy will then be

$$j_U = - n\mu_e(E_c-\varepsilon_F+\frac{3}{2} k_B T)E+p\mu_h(\varepsilon_F-E_v+\frac{3}{2}k_B T)E \quad (4)$$

Dividing by the flow of charge given by (1), we get Peltier's coefficient

$$\Pi = \frac{j_U}{j_q} = \frac{[M(f-1)+f]E_g - \frac{3}{2}(M-1)k_B T}{(M+1)e} \quad (5)$$

where $M = n\mu_e / p\mu_h$ and use was made of eq. (3). In the intrinsic temperature range we have $n=p$ and therefore $M = \mu_e / \mu_h$.

At least for moderate intervals of temperature, M, f and E_g can be assumed as independent of T and we can write

$$\Pi = A + BT \quad (6)$$

the constants A and B being

$$A = \left(f - \frac{M}{M+1}\right) \frac{E_g}{e} \qquad B = - \frac{3}{2} \frac{M-1}{M+1} \frac{k_B}{e} \quad (7)$$

The presence of a thermal gradient will produce, within the semiconductor, the electric field

$$E = - \text{grad } V = \frac{\Pi}{T} \text{grad } T \quad (8)$$

where V is the electrostatic potential.

The scalar product of (8) by the elementary length gives

$$- dV = \frac{\Pi}{T} dT \quad (9)$$

or, using (6),

$$dV = - \left(\frac{A}{T} + B \right) dT \quad (10)$$

which integrates into

$$V - V_0 = - A \ln \frac{T}{T_0} - B(T - T_0) \quad (11)$$

V_0 and T_0 being integration constants that make $V = V_0$ when $T = T_0$.

3. THOMAS-FERMI APPROXIMATION

As between E_v and ε_F (Fig. 1) there are no allowed orbitals, E_v can be computed by the expression, which gives ε_F in metals,

$$E_v = \frac{h^2}{8\pi^2 m} (3\pi^2 n)^{2/3} \quad (12)$$

Here h is Planck's constant, m is the electronic mass and n is now the free electron concentration.

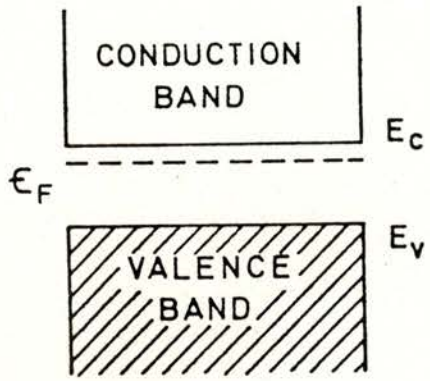

Fig. 1 - Energy bands of a semiconductor. At absolute zero, the conduction band is empty and the valence band completely filled.

The chemical potential can be defined as

$$U = \varepsilon_F(n) - e(V - V_0) \qquad (13)$$

where V_0 is a constant to be determined. If n_0 is the concentration when $V = V_0$ we have then $U = \varepsilon_F(n_0)$.

According to the Thomas-Fermi approximation, the chemical potential must be constant for any part of a substance in thermal and diffusive equilibrium [5]. Therefore

$$\varepsilon_F(n) - e(V - V_0) = \varepsilon_F(n_0) \qquad (14)$$

Expanding $\varepsilon_F(n)$ in a Taylor series, we get

$$\varepsilon_F(n) = \varepsilon_F(n_0) + \left(\frac{d\varepsilon_F}{dn}\right)_{n=n_0} (n - n_0) + \dots \quad (15)$$

Substituting in (14) and neglecting powers of $n - n_0$ of second or higher order, we obtain

$$n - n_0 = \frac{e}{(d\varepsilon_F / dn)_{n=n_0}} (V - V_0) \qquad (16)$$

Using (3), with $f E_g$ constant,

$$\frac{d\varepsilon_F}{dn} = \frac{dE_v}{dn} \qquad (17)$$

Now, taking logarithms of both members of (12) and differentiating, we have

$$\left(\frac{d\varepsilon_F}{dn}\right)_{n=n_0} = \frac{2E_v(n_0)}{3n_0} \qquad (18)$$

and with this value (16) becomes

$$n - n_0 = \frac{3en_0}{2E_v(n_0)} (V - V_0) \qquad (19)$$

Multiplying by $-e$ and using (11), we obtain finally the density of charge

$$\rho = -e(n - n_0) = \frac{3e^2 n_0}{2E_v(n_0)} [A \ln \frac{T}{T_0} + B(T - T_0)] \qquad (20)$$

n_0 and T_0 being constants to be determined.

4. MODEL OF THE CORE

For the present computations we shall use the PREM model [6], which has the following dimensions:-

Radius of the inner core:	R_1 =1221.5 km
Radius of the outer core:	R_2 = 3480 km

The density (in g/cm^3) of the outer core is described by the polynomial

$$d = 12.5815 - 1.2638\,\frac{r}{R} - 3.6426\left(\frac{r}{R}\right)^2 - 5.5281\left(\frac{r}{R}\right)^3 \qquad (21)$$

r being the radius at each level and R = 6371 km being the mean radius of the Earth.

As what concerns chemical composition, we extended Ringwood's ideas [2] by assuming that both the solid inner core and the molten outer core are formed by the oxide $Fe_{0.9}Ni_{0.1}O_{0.5}$. The corresponding kg-molecule is $M_{mol} = 64.125$ kg.

This material is certainly a semiconductor, to which we attributed tentatively the following properties:-

$$M = 1.2 \qquad f = 0.8 \qquad E_g = 0.4\ \text{eV}$$

It is a p-type semiconductor (because Peltier's coefficient is positive). Within a first approximation, we assume that eq. (20) also applies (with the same parameters) to the molten core material.

5. ROTATION AND TEMPERATURE

The actual rotation of the Earth's mantle must convey the whole core; but, as the inner core is separated from the mantle by a thick liquid layer, its rotation could be slightly advanced or retarded in relation to the mantle.

It was previously proposed [3] that the relative angular velocity of the inner core is expressed in the westward drift of the geomagnetic field. The determination of this drift is somewhat difficult; an approximate mean value is $\omega_1 = -1.0*10^{-10}$ rad/s [7,8].

The relative angular velocity ω must decrease in the molten outer core, vanishing at the mantle-core boundary. The variation is given approximately by the equations

$$\frac{\omega}{\omega_1} = 1 \qquad \text{for} \quad r < R_1$$

$$\frac{\omega}{\omega_1} = \frac{R_1^\beta}{R_2^\beta - R_1^\beta}\left(\frac{R_2^\beta}{r^\beta} - 1\right) \qquad \text{for } R_1 < r < R_1 \qquad (22)$$

β being a function of the viscosity of the molten material [9]. We made $\beta = 7.39$ which corresponds to a viscosity of about 10^4 Pa.s. A graph of eq. (22) is shown in Fig. 2.

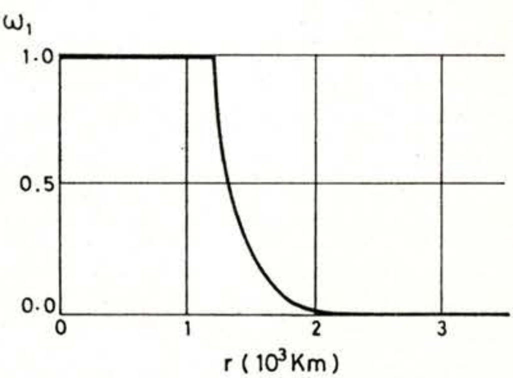

Fig. 2 - Variation of angular velocity in the Earth's core (relative to the mantle). Viscosity of the outer core was assumed to be about 10^4 Pa.s.

In this model, the temperature at the boundary between inner and outer core

must be the melting point of the material. We are using the value of 5000 K, as proposed by Poirier [10].
As to what concerns the thermal gradient, it is believed that the content of radioactive elements (assumed to be similar to the content in iron meteorites) is too small for producing an adiabatic gradient. If heat is transmitted only by conduction, the temperature must satisfy the equation

$$\text{lap}\, T = -\frac{H}{K} \qquad (23)$$

H being the radioactive heat production per unit volume and k the thermal conductivity.
Assuming spherical symmetry, and constancy of H and k throughout the core, integration of (23) gives

$$T = 5000 + \frac{H}{6k}\left(R_1^2 - r^2\right) \qquad (24)$$

Here $T = 5000$ K for $r = R_1$, as initially postulated.

6. GENERATION OF THE GEOMAGNETIC FIELD

As the Earth's core, taken as a whole, is thought to be in a neutral electrical state, the total charge must be zero, i.e.

$$4\pi \int_0^{R_2} \rho r^2 dr = 0 \qquad (25)$$

Noting that for H/k small we have approximately $\ln(T/T_0) = (H/6k)(r_0^2 - r^2)/5000$ (with $r = r_0$ for $T = T_0$), and substituting from (20) we get

$$\int_0^{R_2} \left(r_0^2 - r^2\right) r^2 dr = 0 \qquad (26)$$

giving (for the assumed model) $r_0^2 = 3R_2^2/5$. The concentration n_0 will be

$$n_o = \frac{Nd_0}{M_{mol}} \qquad (27)$$

where N is the Avogadro constant and d_0 a density obtained from (21). We get r_0 = 2695.6 km, d_0 = 10.99 g/cm^3, $n_0 = 1.032 * 10^{29}$ m^{-3} and $E_v(n_0)$ = 8.02 eV; and we now have all the constants which appear in eq. (20), except for H/k which is included in T.

The magnetic moment of the Earth is, approximately,

$$M_{mgn} = \frac{4\pi}{3} \int_0^{R_2} \rho \omega r^4 dr \qquad (28)$$

and can be computed by direct substitution of (20) and (22) (using for $\ln(T/T_0)$ the approximation indicated above).
The value of the moment is $M_{mgn} = -7.94 * 10^{22}$ Am^2, referred to the 1975 geomagnetic field [11]. The factor H/k of eq. (24) can be chosen for having this value of M_{mgn}.

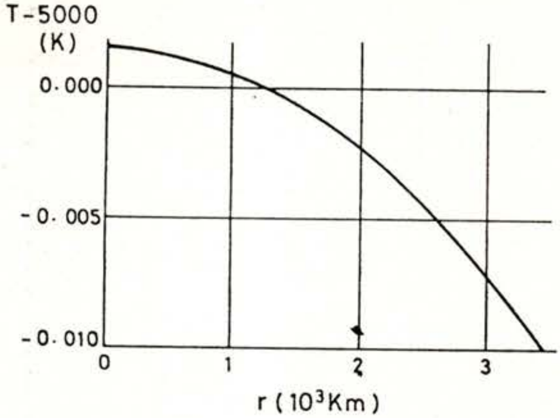

Fig. 3 - Variation of temperature in the Earth's core, with H/k = $5.66*10^{-9}$ K/km^2. Heat transmission is supposed to be by conduction only.

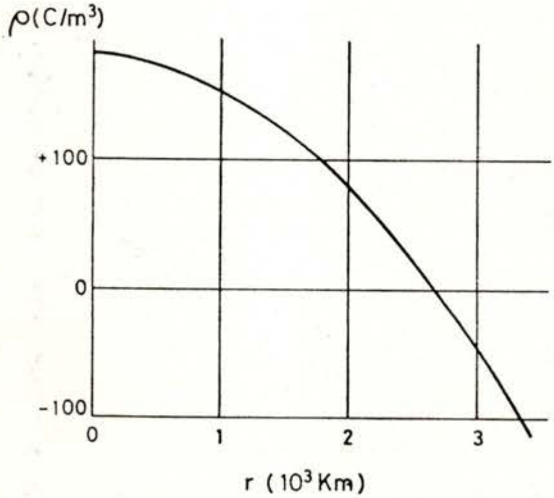

Fig. 4 - Variation of electric charge density in the Earth's core. The values were obtained as described in the text.

For the material described in section 4, we obtained H/k = $5.66*10^{-9}$ K/km^2, which is a value in the magnitude range of iron meteorites. The corresponding temperatures are shown in Fig. 3 and the charge densities in Fig. 4.

These results indicate that the proposed mechanism is a good possibility, but the actual composition and properties of the core model have certainly to be improved by laboratory research.

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