MESONIC COUPLINGS IN THE NAMBU-JONA-LASINIO MODEL. THE $\sigma \rightarrow \pi\pi$ DECAY

M. C. RUIVO

Centro de Física Teórica Departamento de Física da Universidade de Coímbra, 3000 Coímbra
Portugal

ABSTRACT-A technique to calculate the coupling between mesons, in the framework of the Nambu-Jona-Lasinio model is proposed. The approach is based on the Time Dependent Hartree-Fock Theory and consists of a boson expansion including appropriate anharmonic terms. The technique is applied to the calculation of $g_{\sigma\pi\pi}$, for the bound state solution as well as for the discretized solutions of the $q\bar{q}$ continuum. The physical meaning of these solutions is discussed.

1. INTRODUCTION

The interpretation of the mesonic spectrum in terms of the underlying dynamics of strong interactions is nowadays an important issue in particle physics. Both the difficulties of experimental identification of those mesons and the controversy about their quark structure show that many questions remain open [1],[2]. Different approaches have focused on the scalar meson problem. Besides the conventional description of those mesons as $q\bar{q}$ states, which does not allow to fit all the data available, interpretations of their structure as multiquark states, glueballs or suitable combinations of those states have been explored [2],[3],[4],[5],[6],[7],[8]. The work here reported is part of a program of investigation of meson properties within the framework of the Nambu-Jona-Lasinio (NJL) model [9] where the mesons are taken as $q\bar{q}$ excitations. The approach, based on the conventional Time Dependent Hartree-Fock (TDHF) formalism, as developed to deal with non-relativistic nuclear structure situations, explores the analogy between the model and a many-body system of non-relativistic fermions [10]. The aim of the present work is, in the first place, to present a bosonization technique, which takes into account anharmonic terms responsible for the couplings between mesons. The technique is here used to calculate the decay of the scalar-isoscalar mesons into two pions, within the framework of a $SU(N_f = 2)$ NJL model and applied to the bound state solution as well as to the discretized solutions of the

$q\bar{q}$ continuum. We hope to provide, in this way, some physical insight on the discretization of the continuum. This version of the model is, certainly, too simple to provide a realistic description of the scalar mesons and we do not expect, in this preliminary work, to obtain accurate quantitative results. The technique proposed might, however, be applied to more sophisticated schemes and to the decay of other mesons.

The NJL model is described by an effective Lagrangian of relativistic fermions interacting through a two-body contact force. The gluonic degrees of freedom are assumed to be frozen. The model, which incorporates the basic symmetries of QCD and satisfies the relevant current algebra relations, provides an useful tool to investigate the low energy region of the hadronic spectrum. For zero current quark masses it allows for the description of the mechanism of dynamical chiral symmetry breaking, which leads to a vacuum of $q\bar{q}$ condensates associated with the emergence of massless collective excitations of $q\bar{q}$ with the quantum numbers of pseudoscalar isovector mesons (the Goldstone pions) and with the occurrence of a mass of dynamical origin for the constituent quarks. Excitations of $q\bar{q}$ states, with proper quantum numbers of mesons, may be extracted from the new vacuum.

Although the pion sector is well described within the original versions of the model, problems with the description of the other mesons lead, for instance, to the construction of generalized versions [11,12]. Interest in the excitation modes of the NJL model was restricted, until recently, to bound states. Although the existence of the modes of the continuum was recognized, they were commonly disregarded. As a matter of fact, unless a confining mechanism is implemented in the model, unbound states would decay into $q\bar{q}$ pairs, being considered as unphysical. However, recently these modes have been object of interest [13,14,15,16]. In [13] a method for obtaining the solutions of the NJL model by means of a polynomial ansatz was proposed. This method leads to a discretization of the continuum and is equivalent to introducing a constraint on the $q\bar{q}$ relative motion. This might be faced as a modification of the original NJL model, in which effects of a confining mechanism are incorporated. The same mechanism was recently implemented in the framework of an extended NJL model [14]. The results obtained for the meson spectrum are in good qualitative agreement with experience, providing, therefore, support for the interpretation of the mesonic excitations of the continuum.

The calculation of the decay amplitudes of those modes is an essential piece of information for a possible identification with physical resonances. In the present work we propose a method to calculate these quantities.

This investigation is carried out within the framework of a TDHF formalism for the NJL model. Previously, this formalism was implemented in the small amplitude limit of the mean field description leading to linearized equations of motion for the excitation modes, equivalent to the Random Phase Approximation (RPA) equations [10]. In this approximation the coupling between the normal modes is neglected but the effects of such couplings might be taken into account through adequate inclusion of anharmonic terms. The bosonization technique for calculating the $\sigma\pi\pi$ coupling is an extension of the previous treatment, consisting in enlarging the expansion of the effective Lagrangian in order to include anharmonic terms associated with the $\sigma\pi\pi$ coupling [17]. An analogous role of anharmonicities in the damping of giant resonances of many-body systems is considered in [18].

We start with a brief review of the model and formalism and of the concepts involved in the discretization of the continuum. Then we present the description of the method for calculating $g_{\sigma\pi\pi}$. The calculation of the decay amplitude for different solutions of scalar modes follows straightforwardly. Finally, the results are discussed.

2 DESCRIPTION OF THE METHOD

2.1 Review of the Formalism.

The dynamics of a many-body interacting system within TDHF formalism is described by a Hamiltonian of the generic form:

$$H = \sum_{i=1}^{N} t(i) + 1/2 \sum_{i \neq j} v(i,j) . \qquad (1)$$

We write, therefore the Hamiltonian of the NJL model as:

$$H = \sum_{i=1}^{N} [\gamma_5 (i)\sigma(i).\mathbf{p}_i + \beta(i)m_0]$$

$$-g\sum_{i \neq j} \delta (\mathbf{x}_i - \mathbf{x}_j)\, \beta(i)\, \beta(j)\, [I - \gamma_5 (i)\, \gamma_5 (j)\, \tau(i).\tau(j)] \qquad (2)$$

where m_0 is a small current quark mass ($m_u = m_d = m_0$), the τ^a ($a = 1,2,3$) are the $SU(N_f = 2)$ generators, β, γ_5 and $\gamma_5\sigma$ are Dirac matrices and g is the coupling constant. The Hamiltonian (2) is left invariant under a chiral rotation in the γ_5 -isospin space, if $m_0 = 0$. The vacuum state is described by a Slater determinant of negative energy states, $|\phi_0\rangle$, with momentum lower than a cutoff Λ, or, equivalently, in terms of the HF density matrix:

$$\rho_0 = 1/2\, [I - (\beta M + \gamma_5\sigma.\mathbf{p})/E]\, \theta\, (\Lambda^2 - p^2) , \qquad (3)$$

where $E=(p^2+M^2)^{1/2}$ and M is a variational parameter interpreted as the mass of the constituent quarks, which is given by the gap equation:

$$1 - m_0/M = 26g \sum_{p} \frac{\theta(\Lambda^2 - \pi^2)}{E} . \qquad (4)$$

This equation was obtained by minimizing with respect to M the functional of the energy:

$$\varepsilon[\rho] = \mathrm{tr}_1\, \rho(1)\, t(1) + \frac{1}{2} \mathrm{tr}_1 \mathrm{tr}_2\, \rho(1)\rho(2) v^A(12), \tag{5}$$

where v^A (12) is the antisymmetrized two-body interaction.

Deviations from the state of equilibrium lead to a deformed state $|\phi\rangle = U\, |\phi_0\rangle$, which may also be described in terms of the general density matrix $\rho = U\, \rho_0\, U^+$, where U is an unitary time-dependent operator. The time evolution of the system may be derived from the Lagrangian:

$$\underline{\underline{L}}^{(2)} = i\,\mathrm{tr}\,(\dot{U}\dot{\rho}_0 U^+) - \varepsilon[\rho]. \tag{6}$$

Choosing $U = \exp(iS)$, where S is a hermitian single particle time-dependent operator, and assuming that the fluctuations around the equilibrium configuration are small, the Lagrangian may be expanded up to second order in S, leading to

$$\underline{\underline{L}}^{(2)} = \frac{i}{2} \langle\phi_0\, |[S,\dot{S}]|\phi_0\rangle - \frac{1}{2} \langle\phi_0\, |[S\, ,[H,S]]|\phi_0\rangle \tag{7}$$

By making use of the action principle, performing arbitrary variations with respect to the variational functions contained in S and assuming harmonic dependence on time, one obtains homogenous linearized equations of motion, equivalent to the RPA equations.

2.2 Discretization of the continuum

The mesonic excitations of the vacuum below the $q\bar{q}$ threshold (E_{th}=2M), investigated in [10], are solutions of exact RPA equations. The same treatment might easily be used to explore the region above the threshold ($q\bar{q}$ continuum). This region was, until recently, considered as not worthy of interest, due to the lack of confining mechanism in the model. However, as it is well-known, there is some strength which is not exhausted by the bound state solutions and is localized in the continuum, as explained in [13]. Information concerning meson properties should, therefore, lie also in the continuum of the model. The question is how this information can be extracted.

This region was studied, within the framework of the formalism described above, using a technique which discretizes the continuum [13]. The basis of the technique is very simple and consists in replacing the variational functions of the generators of the fluctuations, which are generic functions of **p**, by low order of polynomials. By using polynomials of the form $a + bp^2$, two discrete solutions of the RPA equations are obtained: one replaces the bound state solution and the other the continuum. The constraint imposed in the momentum space reduces the infinite number of continuum modes to one single mode, in an analogous way to what would be expected from a confining mechanism. In the modified model large values of the $q\bar{q}$ distance are forbidden. The masses obtained in this way for the low-lying mesons are in good agreement with experiment. The lowest pseudoscalar-isoscalar mode always appears at zero energy, in the chiral limit, and small deviations of the current algebra relations are obtained.

An exact and covariant treatment of the $q\bar{q}$ continuum of a $SU(N_f = 2)$ NJL model is reported in [15]. The masses of the mesons are identified with the center of gravity of the strength distribution. In [16] the continuum modes are also studied in the context of a generalized NJL model.

All these approaches call for the attention of the continuum modes and predict values for their masses. In order to clarify its physical meaning one should look at their decays. One could regard the decay of the continuum modes through two different mechanisms:

-The Landau damping of the exact solutions, which means that the collective $q\bar{q}$ modes spread its strength over a multitude of continuum normal modes and lose their identity due to interference effects. This does not correspond, in the original NJL model, to the true physical decay. This mechanism is prevented to occur in the modified model by our choice of the generating functions.

-The two-body damping of the discretized solutions in few normal modes, which we regard as the true physical decay.

In order to calculate the decay amplitude of the resonances in specific channels one should implement the mechanism analogous to the two-body damping in many-body systems. This is achieved by performing an adequate bosonization of the original Hamiltonian, including anharmonic terms which are responsible for the decay of the normal modes.

The bosonization technique described below is applied to the calculation of the decay of the exact bound state and of the discretized solutions of scalar-isoscalar mesons into $\pi\pi$. Convenient adaptations of the same technique might be used to calculate other decays.

This will also allow to clarify the meaning of the discretization technique, which should by no means be regarded as an approximation in order to avoid the exact RPA treatment (which, anyway, does not present particular difficulties) but as a device to incorporate effects of confinement.

2.3 Bosonization technique and calculation of $g_{\sigma\pi\pi}$

Bosonization is nothing more than an identification of canonical coordinates. As is well-known, the RPA approximation is the lowest order of a boson expansion which maps a fermion subspace into a boson subspace, the HF vacuum, $|\phi_0\rangle$, being mapped into the RPA vacuum, $|\,\rangle$, and the fermion operators into boson operators. In the present case we are interested in an expansion in boson operators (canonical coordinates) up to third order. We start by expanding the functional of the energy $\varepsilon[\rho]$ in powers of S up to this order:

$$\varepsilon[\rho] = \langle\phi|H|\phi\rangle \approx \langle\phi_0|H|\phi_0\rangle + \frac{1}{2}\langle\phi_0|[S,[H,S]]|\phi_0\rangle + \frac{i}{3!}\langle\phi_0|[S,[S,[S,H]]]|\phi_0\rangle, \quad (8)$$

In order to make a connection with the RPA formalism it is convenient to write the generator of the fluctuations of the vacuum as:

$$S = \sum_\tau (\alpha_\tau \Theta_\tau e^{i\omega_\tau t} + \alpha_\tau^* \Theta_\tau^+ e^{-i\omega_\tau t}) \quad (9)$$

where ω_τ is the frequency of a generic mode of excitation τ and Θ_τ, (Θ_τ^+) are one-body fermion operators. These operators may be normalized by imposing the condition:

$$\langle\phi_0|[\Theta_\tau, \Theta_\tau^+]|\phi_0\rangle = 1, \quad (10)$$

The variables α_τ and α_τ^* are canonical coordinates. Expanded in these variables, the harmonic terms describe the energy of the modes and the anharmonic (third order terms) describe their decays. The harmonic term of the functional of the energy can be written as:

$$\varepsilon^{(2)}(\alpha_\tau^*, \alpha_\tau) = \omega_\tau \alpha_\tau^* \alpha_\tau \quad , \quad (11)$$

with:

$$\omega_\tau = \langle\phi_0|[\Theta_\tau, [H, \Theta_\tau^+]]|\phi_0\rangle, \quad (12)$$

The harmonic RPA effective Hamiltonian may, therefore, be written as

$$\hat{H} = \omega_\tau A_\tau^+ A_\tau, \quad (13)$$

where $A_\tau^+ (A_\tau)$ are boson operators.

In order to calculate $g_{\sigma\pi\pi}$ we need the appropriate anharmonic perturbation. Writing $S = S_\sigma + S_\pi$, and taking advantage of the form of the generator S (9), we easily obtain the component of the third order term, relevant for describing the process $\sigma-\pi\pi$:

$$\varepsilon_{\sigma\pi\pi} = \lambda_{\sigma\pi\pi}(\alpha_\sigma \alpha_\pi^* \alpha_\pi^* + \alpha_\pi \alpha_\pi \alpha_\sigma^*), \quad (14)$$

with:

$$\lambda_{\sigma\pi\pi} = \frac{i}{3!}\langle\phi_0|[\Theta_\sigma,[\Theta_\pi^+,[\Theta_\pi^+,H]]] + [\Theta_\pi^+,[\Theta_\sigma,[\Theta_\pi^+,H]]] + [\Theta_\pi^+,[\Theta_\pi^+,[\Theta_\sigma,H]]]|\phi_0\rangle, \quad (15)$$

By definition, $g_{\sigma\pi\pi}$ is given by:

$$g_{\sigma\pi\pi} = \lambda_{\sigma\pi\pi}\sqrt{2\omega_\sigma}\,(2\omega_\pi), \quad (16)$$

In order to describe a process in which one σ at rest decays into two pions with opposite momenta, one should have a perturbation Hamiltonian of the form:

$$W = \sum_{\mathbf{k}} \frac{1}{2}\lambda_{\sigma\pi\pi}(\mathbf{k})\,(B^+ A_{\mathbf{k}} A_{-\mathbf{k}} + A_{-\mathbf{k}}^+ A_{\mathbf{k}}^+ B), \quad (17)$$

where W is the effective interaction RPA Hamiltonian and $B^+(B)$, $A_{-\mathbf{k}}^+(A_{\mathbf{k}})$ are creation (annihilation) operators, respectively for σ and π.

Here we take the following approximation:

$$\lambda_{\sigma\pi\pi}(\mathbf{k}) \simeq \frac{g_{\sigma\pi\pi}}{\sqrt{2\omega_\sigma}\,(2\omega_\pi(\mathbf{k}))} \quad . \qquad (18)$$

The transition amplitude is easily obtained through the Fermi Golden rule. In the chiral limit we have:

$$\Gamma_{\sigma\pi\pi} = \frac{3|g_{\sigma\pi\pi}|^2}{16\pi\omega_o}\left(1 - \left(\frac{2\omega_\pi}{\omega_\sigma}\right)^2\right)^{1/2} \qquad (19)$$

where the factor 3 comes from isospin degeneracy. The calculation of $\lambda_{\sigma\pi\pi}$ from (15) is straightforward. Bearing in mind that, within our formalism the pseudoscalar-isovector modes and the scalar-isoscalar modes are described, respectively, by the generators:

$$S_\pi^a = [\gamma_5 L_1 + i\,\beta\gamma_5 L_2]\tau^a\,, \quad a = 1,2,3 \qquad (20)$$

$$S_\sigma = \gamma_5\,\sigma.\pi F_1 + i\,\beta\gamma_5\,\sigma.\pi F_2\;, \qquad (21)$$

where L_i and F_i (i=1,2) are variational functions depending on p^2 and t, one obtains:

$$\lambda_{\sigma\pi\pi} = 48\,M{*}26\,g\left[\sum_p \frac{p^2F_2}{E}\Theta|\Lambda^2-p^2| \sum_p \frac{L_1^{*2}+L_2^{*2}}{E}\Theta|\Lambda^2-p^2| - \sum_p \frac{L_1^*}{E}\Theta|\Lambda^2-p^2| \sum_p \frac{p^2}{E}(F_2L_1^* + F_1L_2^*)\,\Theta|\Lambda^2-p^2|\right] \qquad (22)$$

The variational functions L_i and F_i were taken as the eigenvectors of the RPA equations of previous works [10,13]. The numerical values were calculated in the chiral limit.

3. DISCUSSION OF THE RESULTS

We show in Table 1. numerical values for ω_σ, $g_{\sigma\pi\pi}$ and $\Gamma_{\sigma\pi\pi}$, obtained, respectively, for the bound state solution and for the discretized solutions of σ decaying into two pions for $\lambda = 2M$ and different values of M. These input parameters obey the self-consistent equation and were chosen so that the quantities $<\bar{\Psi}\Psi>$ and f_π are in the range of its empirical values (see [10]). The numerical values shown are the result of an improved calculation, in relation to those included in [17]. The calculation of $g_{\sigma\pi\pi}$ for $\Omega_\sigma = 2M$ is useful as a check to our method. This decay has already been calculated by other authors using different techniques [19],[20] The value obtained for the decay width does not allow to identify this solution with an established physical particle. The problem of assigning a physical meaning to this solution is related with the controversial existence of a scalar-isoscalar meson with a mass below

700 MeV. The behaviour of this solution with temperature [19] and density [20] was already discussed.

Table 1.

	Conv. RPA			Const. RPA		
M (MeV)	ω_σ (MeV)	$\|g_{\sigma\pi\pi}\|$ (MeV)	$\Gamma_{\sigma\pi\pi}$ (MeV)	ω_σ (MeV)	$\|g_{\sigma\pi\pi}\|$ (MeV)	$\Gamma_{\sigma\pi\pi}$ (MeV)
335	670	2861	660	716	3986	1184
				1269	1350	84
350	700	2989	693	748	4112	1242
				1326	1410	87

Table 1. ω_σ, $|g_{\sigma\pi\pi}|$ and $\Gamma_{\sigma\pi\pi}$ for solutions obtained with the conventional RPA and the constrained RPA.

The results for the couplings and decay widths of the modes obtained through the discretization technique are new and require further consideration.

As it was already mentioned, the polynomial ansatz leads to two modes, one replacing the exact bound state solution and the other replacing the continuum modes. The results obtained for the masses and decay widths support this interpretation. The comments made before concerning the solution $\omega_\sigma = 2M$ apply to the first mode. The value obtained for the decay width is in agreement with the lack of experimental evidence for a resonant behaviour in the scalar-isoscalar channel in the region of 600 - 700 MeV [1]. The second mode might be interpreted as a low-lying scalar-isoscalar meson. The two lowest established scalar-isoscalar resonances are the $f_0(975)$ with decay amplitude Γ = 34 MeV (75% into $\pi\pi$ and 25% into $K\bar{K}$) and the $f_0(1400)$ with Γ = 150 - 400 MeV ($\approx$ 90% into $\pi\pi$ and $\approx$ 10% into $K\bar{K}$) [1]. Although a $SU(N_f = 2)$ model cannot provide a realistic description any of those mesons, which have a component of strangeness, the $f_0(975)$ is more unlikely described by this version of the model (even at a qualitative level). A description of this meson as a $q\bar{q}$ state, even allowing for flavour mixing, is often considered unsuitable to account for its properties (namely its branching ratio into $K\bar{K}$ (25%), in spite of the small phase space available). Interpretations of this meson as a $K\bar{K}$ molecule seems more appropriate, while the $f_0(1400)$ is more commonly interpreted as a true resonance [1,6,21]. In view of this situation it seems reasonable to compare the present results for the second mode to the $f_0(1400)$.

Moreover, the numerical values obtained provide support to this comparison. We notice, however, that our value for the decay width is low compared to the estimated experimental value of the $f_0(1400)$.

The present results should be regarded as essentially qualitative. The main point is that we show that it is possible to implement a mechanism through which the discretized modes of the continuum can decay in specific meson channels. The bosonization technique, together with the polynomial ansatz, makes possible a consistent treatment of the modes of the continuum. The investigation of the effect of giving to the scalar mesons a more complex structure in the framework of a generalized NJL model and the improvement of the present technique, are feasible within the TDHF formalism. Research along these lines is being carried out.

ACKNOWLEDGEMENTS

I would like to thank J. da Providência, M. K. Banerjee, C. A. de Sousa, B. Hiller and A. H. Blin for helpful discussions. This work was supported by Instituto Nacional de Investigação Científica.

REFERENCES

[1] Particle Data Group, Phys. Lett. **B239**, 1 (1990).

[2] Diekmann, B., Phys. Rep. **159**, 99 (1988).

[3] Sharpe, S. R., Jaffe, R. L. and Pennington, M. R., Phys. Rev. **D30**, 1013 (1984).

[4] Teshima, T. and Oneda,S., Phys. Rev. **D33**, 1974 (1986).

[5] Estabrooks, P., Phys. Rev. **D19**, 2678 (1979).

[6] Weinstein, J. and Isgur, N., Phys. Rev. **D27**, 588 (1983) and Phys. Rev. **D41**, 2236 (1990).

[7] Mennessier, G., Narison, S. and Paver, N., Phys. Lett. **B158**, 153 (1985).

[8] Au, K. L., Morgan, D. and Pennington, M. R., Phys. Rev. **D35**, 1633 (1987) and Phys. Lett. **B167**, 229 (1986).

[9] Nambu, Y. and Jona-Lasinio, G., Phys. Rev. **122**, 345 (1961) and Phys. Rev. **124**, 246 (1961).

[10] da Providência, J., Ruivo, M. C. and de Sousa, C. A., Phys. Rev. **D36**, 1882 (1987) and Phys. Rev. **D38**, 2646 (1988); de Sousa, C. A., Z. Phys. **C43**, 503 (1989).

[11] Ebert, D. and Reinhardt, H., Nucl. Phys. **B271**, 188 (1986).

[12] Bernard, V. and Meissner, U. G., Nucl. Phys., **A489**, 647 (1988).

[13] da Providência, J. and de Sousa, C. A., Phys. Lett. **B237**, 147 (1989).

[14] de Sousa, C. A., Z. Phys. C **49,** 619 (1991)

[15] Blin, A. H., Hiller, B. and da Providência, J., Phys. Lett. **B241**, 1 (1990).

[16] Klimt, S., Lutz, M., Vogl, U. and Weise, W., Nucl. Phys. **A516**, 429 (1990).

[17] Ruivo, M. C., Europhys. Lett., **15**, 139 (1991) [18] da Providência, J., Europhys. Lett. **4**, 789 (1987).

[19] Hatsuda, T. and Kunihiro, T., Phys. Lett. **B185**, 304 (1987) and Prog. Theor. Phys. **91**, 248 (1987).

[20] Bernard, V., Meissner, U. G. and Zahed, I., Phys. Rev. Lett. **59**, 966 (1987).

[21] Lohse, D., Durso, J. W., Holinde, K. and Speth, J., Nucl. Phys. **A516**, 513 (1990).